ACE Answers | Investigation 4

Applications

Answers involving pi were calculated using the π key on a calculator. If students use 3.14 for pi, their answers may vary slightly.

1. a. The shorter cylinder has the greater volume because it has a greater base area (with a longer radius), and the circumference of the longer cylinder is the height of the shorter one.

b. The shorter cylinder has the greater surface area because both cylinders have the same lateral area, but the shorter cylinder has a greater base area.

2. a. Answers may vary. Most people's instinct is to say, "Height is greater than circumference," because the height of the cylinder is more accessible as a linear measurement.

b. If the balls are tightly packed, the height is $3d$, and the circumference is πd. So, the circumference is greater than the height.

3. a. A 1-centimeter layer of sand has a volume of $9\pi \approx 28.3 \text{ cm}^3$.

b. 20 layers

c. $180\pi \approx 565 \text{ cm}^3$

4. $\text{S.A.} = 2\pi(6.5)^2 + 2\pi(6.5)(10) \approx 674 \text{ cm}^2$;
$V = \pi(6.5)^2(10) = 422.5\pi \approx 1{,}327 \text{ cm}^3$

5. $\text{S.A.} = 2\pi(10)^2 + 2\pi(10)(6.5) \approx 1{,}037 \text{ cm}^2$;
$V = \pi(10)^2(6.5) = 650\pi \approx 2{,}042 \text{ cm}^3$

6. a. 1 kilometer of pipeline will hold $\pi(1^2)1{,}000 \approx 3{,}141.6 \text{ m}^3$ of oil.

b. 5,000 km of pipeline will hold about $15{,}708{,}000 \text{ m}^3$ of oil.

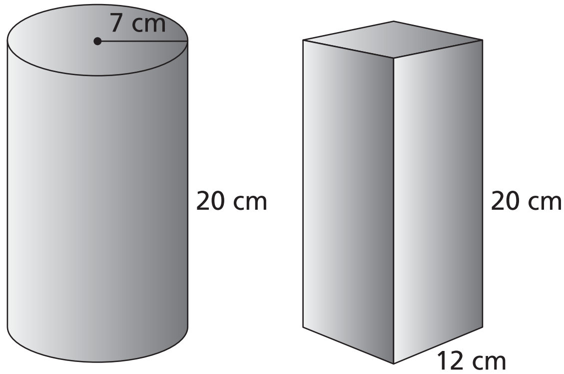
7. a. $V = 54\pi \approx 170 \text{ cm}^3$

b. $\text{S.A.} = 54\pi \approx 170 \text{ cm}^2$

8. a. Volume of the cylinder is $\pi(3^2)10 \approx 283 \text{ cm}^3$; Volume of the square prism is 180 cm^3. (**Hint:** Think of the square base as equal in area to a sum of 2 or 4 triangles or one 3 cm $\times$ 6 cm rectangle.)

b. The ratio of cylinder volume to prism volume is $\pi : 2 \approx 1.57$.

9. a. Sample sketches:



b. For the cylinder, the volume is $980\pi \approx 3{,}079 \text{ cm}^3$; the surface area is $329\pi \approx 1{,}034 \text{ cm}^2$. For the square prism, the volume is $2{,}880 \text{ cm}^3$; the surface area is $1{,}104 \text{ cm}^2$. (**Note:** Neither surface area includes a top face.)

c. The cylinder holds more popcorn, so it costs more to fill. The cylinder also has less packaging and therefore costs less to buy. Additional information on the cost per square centimeter of making boxes and the cost per cubic centimeter of popcorn would help in making the choice of which box is more economical for the vendor.

10. $V = \left(\frac{1}{3}\right)s^2h$.

11. Volume is 120 cm^3.

12. Volume is 40 cm^3.

13. The cone holds about $36\pi \approx 113 \text{ cm}^3$ and the cylinder $28.125\pi \approx 88 \text{ cm}^3$. The club should choose the cylindrical cup because it costs less to fill.

14. The pyramid holds less frozen yogurt (75 cm^3) than the cube (101 cm^3), so the Mathletes should use the pyramid.

15. The pyramid has a volume 4,000 cm^3, which is 1,000 cm^3 of popcorn per dollar; the cone has a volume of $1{,}000\pi \approx$ 3,142 cm^3, which is about 1,257 cm^3 of popcorn per dollar; the cylinder has a volume of $1{,}280\pi \approx 4{,}021$ cm^3, which is about 1,072 cm^3 of popcorn per dollar; the prism has a volume of 3,600 cm^3, which is 1,029 cm^3 of popcorn per dollar. Thus, the cone has the best value of popcorn per dollar.

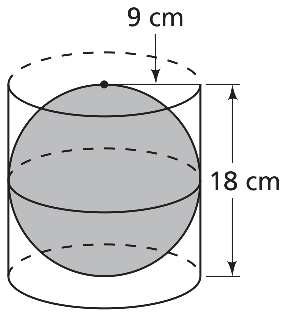
16. Height is approximately 8.8 inches.

17. Height is approximately 26.5 inches.

18. **a.** Prism height is 25 inches.

b. Pyramid height is 75 inches.

19. **a.**



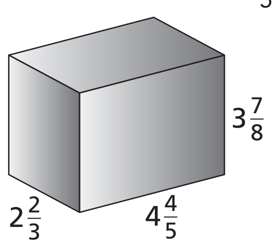
b. Volume of the cylinder is $1{,}458\pi \approx 4{,}580$ cm^3.

c. Volume of the ball is $\left(\frac{4}{3}\right)\pi(9^3) \approx 3{,}054$ cm^3.

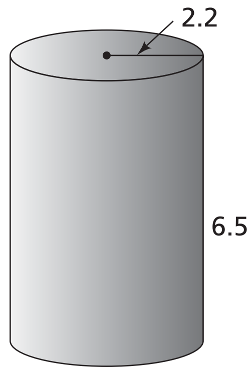
20. $\left(\frac{4}{3}\right)\pi(7.5^3) \approx 1{,}767$ cm^3

21. $\left(\frac{4}{3}\right)\pi(8^3) \approx 2{,}145$ cm^3

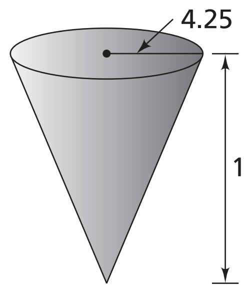
22. This calculation is for the volume of a rectangular prism with the dimensions shown below. The volume is $49\frac{3}{5}$ cubic units.



23. This calculation is for the volume of a cylinder with a radius of 2.2 and a height of 6.5. The volume is approximately 98.8 cubic units.



24. This calculation is for the volume of a cone with a radius of 4.25 and a height of 10. The volume is approximately 189 cubic units.



25. **a.** Sample sketches: (See Figure 1.)

b. Based on the experiment in Problem 4.4, the cylinder will hold the most water and the cone will hold the least water.

c. The volume of the cylinder is $16{,}000\pi \approx 50{,}265$ ft^3; the volume of the sphere is about 33,510 ft^3; the volume of the cone is about 16,755 ft^3. These results confirm the answer to part (b).

26. Radius is approximately 3.9 inches.

27. The scoop has a volume of $\left(\frac{4}{3}\right)\pi$, or about 4.2 in.^{3}; the cone has a volume of $\frac{5\pi}{3}$, or about 5.2 in.^{3}. The cone holds about 1.25 scoops.

28. Filling the glass to one-half inch from the top means the total volume of root beer and ice cream is about 39.27 in.^{3}. Each scoop of ice cream has a volume of about 8.2 in.^{3}, so 3 scoops has a volume of about 24.6 in.^{3}. There will be about 14.7 in.^{3} of root beer in each float, so there will be more ice cream than root beer.

Connections

29. **a.** The rectangular prism has a surface area of 1,128 cm^2; the closed cylinder has a surface area of 290π, or about 911 cm^2. The prism has a greater surface area.

b. The rectangular prism has a volume of 2,016 cm^3; the closed cylinder has a volume of 600π, or about 1,884 cm^3. The prism has a greater volume.

30. The radius will be about 9 feet.

31. Answers will vary. The dimensions of the base could be any pair of numbers whose product is 100.

32. The vertical cut makes a rectangular face on each part; the slant cut makes an oval-shaped face on each part; the horizontal cut makes a circular face on each part.

33. No, each prism could have quite different dimensions for its base as long as the product of the length and width is the same.

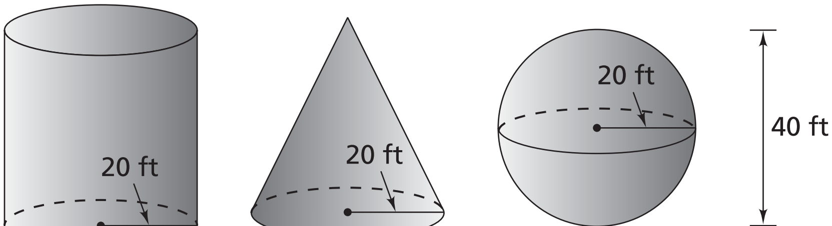
34. Yes, the area of the base is a circle, which depends only on the radius (or, equivalently, the diameter).

35. There are several levels of subtlety that could be applied to this packing problem. Sample strategies for underestimating and overestimating are provided.

An underestimate of the number of cans involves the argument that cans with radii of 3 centimeters (diameters of 6 centimeters) would fit in a rectangular array across the floor of $600 \div 6 = 100$ cans along the 6-meter width and $800 \div 6 = 133$ cans along the 8-meter length. So, one layer would be 13,300 cans. 25 layers of cans 12 cm tall would fit into the 300-cm-high room. 25 layers of 13,300 cans is a total of about 332,500 cans. Since the cans could be arranged more compactly than the simple rectangular lattice that this calculation assumes, the actual capacity is probably greater.

Another strategy that overestimates packing density is to find the volume of each can and the volume of the room and divide. Each can has a volume of 108π cm^3, and the volume of the room is $600(800)(300) = 144{,}000{,}000$ cm^3. Dividing the volume of

Figure 1

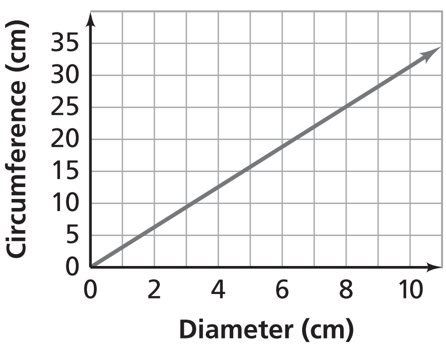


the room by the volume of each can gives an estimated capacity of 424,413 cans. This estimate is greater than the actual number of cans because it does not consider the space between cans.

36. a. **Circle Circumference**

Diameter (cm)	Circumference (cm)
1	3.14
2	6.28
3	9.42
4	12.57
5	15.71
6	18.85
7	21.99
8	25.13
9	28.27
10	31.41

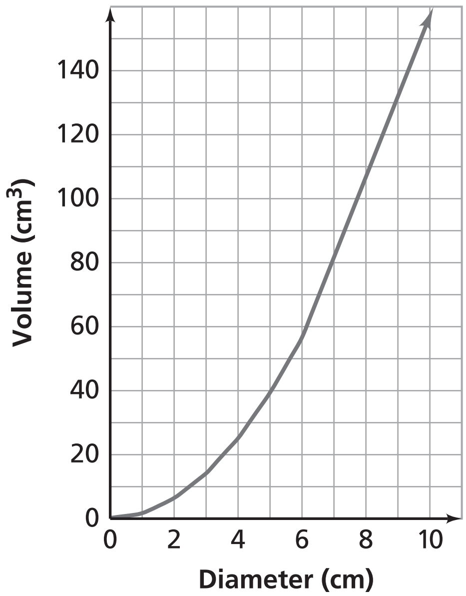
b. A graph of (*diameter, circumference*) data will look like this:



c. **Cylinder Volume**

Diameter (cm)	Circumference (cm)	Volume (cm^3)
1	3.14	1.57
2	6.28	6.28
3	9.42	14.14
4	12.57	25.13
5	15.71	39.27
6	18.85	56.55
7	21.99	76.97
8	25.13	100.53
9	28.27	127.23
10	31.41	157.08

d. A graph of (*diameter, volume*) data will look like this:



e. The (*diameter, circumference*) graph is linear; there is a constant rate of change between the circumference and diameter data. Volume is a quadratic function of diameter for constant height. Students will not yet have encountered quadratic functions. However, they can be expected to notice that the volume increases at an increasing rate as diameter increases. This is shown by the fact that the graph curves upward. In both graphs, the circumference and the volume increase as diameter increases.

37. a. The surface area of the real cylinder is 4 times that of the model.

b. The surface area of the real cylinder is 12.25 times that of the model.

c. The surface area of the real cylinder is f^2 times that of the model.

38. a. The volume is 8 times that of the model.

b. The volume is 42.875 times that of the model.

c. The volume is f^3 times that of the model.

39. The average is 5.03 centimeters.

40. This calculation is for the surface area of a rectangular prism with dimensions $4 \times 8.5 \times 7.25$. In that case, S.A. $= 249.25$.

41. This calculation is equivalent to the previous exercise. It is the surface area of a rectangular prism with dimensions $4 \times 8.5 \times 7.25$. The surface area is again 249.25.

42. This calculation is for the surface area of a cylinder—twice the area of a circular base that has a radius of 4 and the product of the circumference and a height of 8.5 for the lateral surface. The surface area is 100π.

43. The vertical cut exposes two triangles; the slant cut reveals two parabolic regions (students won't know this term, but should be expected to sketch curves that have a parabolic shape); the horizontal cut exposes two circles.

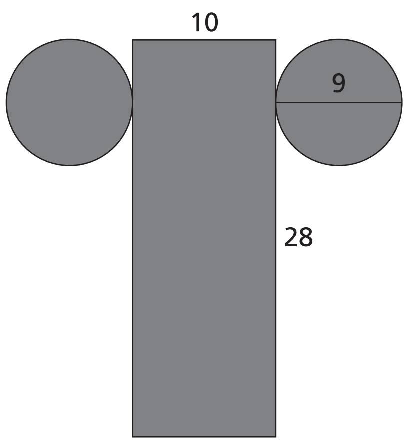
44. In both cases, the cuts expose circles. The cut along the equator reveals circles whose centers are also the center of the sphere. These are called *great circles*.

45. The radius is $\frac{54}{2\pi} \approx 8.6$, so the volume is about $\left(\frac{4}{3}\right)\pi(8.6)^3 \approx 2{,}659$ cm^3.

Extensions

46. Yes, the area of the base will be volume ÷ height. To find the radius, you can divide the area of the base by π and find the square root of that result. Once you know the radius and the height, you can find the circumference of the base, which is also the length of the rectangle in the net.

For example, suppose that the height is 10 and the volume 628. The area of the base is 62.8; the radius squared is $\frac{62.8}{\pi}$, so the radius is about 4.5; the circumference is about 28. The net is shown below.



47. Answers may vary. One way to estimate the volume would be to find the average of two volume calculations, each using one of the two different bases. The estimated volume is $[\pi(4.5^2)14 + \pi(3^2)14] \div 2 \approx 643$ cm^3.

Another strategy (that is not equivalent) is to calculate the volume using the average of the two radii shown. Using this second approach, the estimated volume is $\pi(3.75^2)14 \approx 619$ cm^3.

48. a. Radius is 1cm and height is 10 cm.

b. Volume of empty space is $40 - 10\pi$, or about 8.6 cm^3.

c. Ratio of can volume to box volume is $\frac{\pi}{4}$.

d. i. Dimensions will vary. The ratio will be $\frac{\pi}{4}$.

ii. The ratio will always be $\frac{\pi}{4}$. Since the radius of the cylinder is $\frac{s}{2}$, and both figures have the same height:

$$\frac{\pi\left(\frac{s}{2}\right)^2 h}{s^2 h} = \frac{\pi\frac{s^2}{4}h}{s^2 h} = \frac{\pi}{4}.$$

49. a. The surface area of any such pyramid can be found by finding the sum of the areas of the polygonal faces. The lateral faces are triangles with the same base and height (the slant height drawn from the top of the pyramid perpendicular to an edge of the base). The bases will be different polygons. If the base shape is not a special polygon for which you know an area formula, it is always possible to decompose the polygon into triangles.

b. Based on the earlier experiment with prisms of constant perimeter, students should conclude that the base areas of the pyramids will increase as the number of sides increases. For the triangular faces connecting the base to the top vertex, the heights will also increase as the number of sides increases. While the area of each lateral triangle will decrease, the lateral area and the surface area will be greater.

c. Since the area of the base increases as the number of sides increases, and the height is constant, the volume also increases as the number of sides increases.

d. A cone will have maximum surface area and volume since both measures increase as the number of sides of the base increases. As the number of sides of the base increases, the shape of the pyramid approaches a cylinder. (**Note:** Remind students of Problem 2.1, Question C.)

50. a. Volume of model submarine is $\frac{1}{2} \cdot \frac{4}{3}\pi(3^3) + \pi(3^2)(12) + \frac{1}{3}\pi(3^2)(4) \approx$ 434 in.^{3}.

b. Since 1 inch : 20 feet, the volume of the real submarine is about $20^3(434)$, or 3,472,000 ft^3.

51. a. Area of floor is $\pi(10^2) \approx 314$ ft^2; volume is $\frac{1}{2} \cdot \frac{4}{3}\pi(10^3) \approx 2{,}094$ ft^3.

b. There are many possible combinations of dimensions that will give the same floor area and total volume of inner space if the figure is a rectangular prism. For example, a square floor with sides of approximately 17.7 feet would have the same area. In that case, the height of the room would be about 6.7 feet. In fact, for any combination of dimensions for a rectangular floor with product 314, the height will always be 6.7 feet.

52. a. The largest sphere has a radius of 5 centimeters and a volume of about 524 cm^3.

b. The largest cylinder has a radius of 5 cm, a height of 10 cm, and a volume of about 785 cm^3.

c. The largest cone has a radius of 5 cm, a height of 10 cm, and a volume of about 262 cm^3.

d. The largest pyramid has a square base with edges 10 cm long, a height of 10 cm, and a volume of about 333 cm^3.

53. a. Answers may vary. Every trial should result in about 4 "radius circles."

b. The approximate area of 4 radius circles is $4\pi r^2$.

c. S.A. $= 4\pi r^2$.

d. **Surface Areas of Spheres**

Radius	Surface Area
1	4π
2	16π
3	36π
4	64π
5	100π
10	400π

e. The surface area increases very rapidly as radius increases. In fact, each successive increase of 1 in the radius causes increasingly larger growth in the surface area.

f. When the radius is doubled, the surface area increases by a factor of 4.

g. When the radius is tripled, the surface area increases by a factor of 9.

h. The change in surface area is the square of the scale factor; if the radius is scaled up by a factor f, the surface area is multiplied by f^2. The pattern of growth is the same as it was for the surface areas of 1-2-3 compost boxes and for areas of two-dimensional figures in the Unit *Covering and Surrounding.*