## Applications

Answers involving pi were calculated using the $\pi$ key on a calculator. If students use 3.14 for pi, their answers may vary slightly.

1. diameter; 3 cm
2. circumference; $\approx 5 \mathrm{~cm}$
3. diameter; 7.5 cm
4. radius; 2 cm
5. Exercise 1: Circumference $\approx 9.42 \mathrm{~cm}$.

Exercise 2: Diameter $\approx 1.6 \mathrm{~cm}$.
Exercise 3: Circumference $\approx 23.55 \mathrm{~cm}$.
Exercise 4: Circumference $\approx 12.56 \mathrm{~cm}$.
6. Drawings will vary. Possible drawing:

a. The diameters measure 3.3 cm .
b. Diameters drawn in the same circle have equal measures.
c. Since the circumference is a little more than 3 times the diameter, the circumference is a little more than 3 times 3.3, or about 10 cm .
7. Drawings will vary. Possible drawing:

a. The radii measure 2 cm .
b. Radii drawn in the same circle have equal measures.
c. Since the circumference is a little more than 3 times twice the radius, the circumference is a little more than 3 times 4, or more than 12 cm .
8. Yes. The diameter of a circle is double its radius.
9. Enrique can find half of the diameter to find the radius.
10. C
11. a.

| Size | Diameter <br> (in.) | Radius <br> (in.) | Circum. <br> (in.) | Area <br> (in.2) |
| :--- | :---: | :---: | :---: | ---: |
| Small | 8 | 4 | 25.1 | 50.3 |
| Medium | 10 | 5 | 31.4 | 78.5 |
| Large | 12 | 6 | 37.7 | 113.1 |

b. Jamar is correct; since $\pi$ is approximately $3, \pi r^{2}=\pi\left(\frac{d}{2}\right)^{2} \approx$ $\frac{3 d^{2}}{4}=0.75 d^{2}$. This is an underestimate since $\pi>3$. Note: Students will probably not offer this sort of algebraic reasoning. Instead, they might just check a few cases to see if the numbers turn out the same. It might be good to remind them that such checking gives assurance, but not general proof.
12. Possible answer: The area of a compact disc tells you something about the storage space on the disc.
13. Possible answer: The circumference of a bicycle wheel tells you how far the bike travels in one revolution.
14. Possible answer: The diameter (and related cross-sectional area) of a pipe tells how much water can flow through the pipe.
15. Possible answer: The area of a lawn sprinkler's spray lets you estimate how much of your lawn will get watered at each location the device is used and thus will allow you to estimate how long it will take to water your lawn.
16. Possible answer: The area may help you compare to the pan size specified in a recipe.
17. Answers will vary.
18. circumference: $9 \pi$ in. $\approx 28.3$ in.; area: $4.5^{2} \cdot \pi$ in. $^{2} \approx 63.6 \mathrm{in}^{2}{ }^{2}$
19. radius: 13 in .; circumference: $26 \pi$ in. $\approx$ 81.7 in.; area: $169 \pi$ in. $^{2} \approx 530.9$ in. $^{2}$
20. diameter: 80 ft ; circumference: $80 \pi \mathrm{ft} \approx$ 251.3 ft ; area: $1,600 \pi \mathrm{ft}^{2} \approx 5,026.5 \mathrm{ft}^{2}$
21. a. LP record: radius is 6 in.; circumference is $12 \pi$ in. $\approx 37.7 \mathrm{in}$.; area is $36 \pi$ in. $^{2} \approx$ 113.1 in. ${ }^{2}$

Compact disc: radius is 2.3125 in .; circumference is $4.625 \pi$ in. $\approx 14.5$ in.; area is $(2.3125 \mathrm{in} .)^{2} \cdot \pi$ in. $^{2} \approx 16.8 \mathrm{in} .^{2}$
b. The ratio of the two circumferences is equal to the ratio of their diameters or their radii. In both cases, the ratio is about 2.6 to 1 . So, approximately 2.6 trips around the compact disc would equal 1 trip around the LP record.
c. The ratio of the two areas is approximately 6.73 to 1 , which can be obtained by comparing the results from part (a), by squaring the ratio of the radii of the two discs, or by finding the ratio of the radius squares of the two discs. So, it would take approximately 6.73 compact discs to cover the LP record.
22. If the rectangular lawn is arranged to give maximum area, it will be a square with side lengths 9 m and area $81 \mathrm{~m}^{2}$. The circular exercise space will have diameter $\frac{36}{\pi} \mathrm{~m} \approx 11.5 \mathrm{~m}$ and radius $\approx 5.75 \mathrm{~m}$. This implies an area of about $104 \mathrm{~m}^{2}$. So, the circular exercise run will have a greater area than any rectangle with perimeter 36.
23. $A=50 \times 24+\left(12^{2} \times \pi\right) \div 2 \approx 1,426 \mathrm{ft}^{2}$;
$P=50+50+24+(24 \times \pi) \div 2 \approx 161.7 \mathrm{ft}$
24. Area is $4 \pi \mathrm{~cm}^{2} \approx 12.56 \mathrm{~cm}^{2}$; perimeter is $4 \pi \mathrm{~cm}$.
25. Area is $15 \mathrm{~cm}^{2}$; perimeter is 16 cm .
26. Area $\approx 3.5 \mathrm{~cm}^{2}$; perimeter is $3+1.5 \pi \approx 7.7 \mathrm{~cm}$.
27. Area $\approx 1.75 \mathrm{~cm}^{2}$; perimeter is $3+0.75 \pi \approx 5.4 \mathrm{~cm}$.
28. Area $\approx 5.25 \mathrm{~cm}^{2}$; perimeter is $3+2.25 \pi \approx 10.1 \mathrm{~cm}$.
29. If $b=h=4 \mathrm{~cm}$, area $\approx 8 \mathrm{~cm}^{2}$; perimeter $\approx 13 \mathrm{~cm}$ (isosceles triangle with lengths $4 \mathrm{~cm}, 4.5 \mathrm{~cm}$, and 4.5 cm ).
30. G
31. $C$
32. a. white $9 \pi \mathrm{ft}^{2} \approx 28 \mathrm{ft}^{2}$;
blue $(36-9 \pi) \mathrm{ft}^{2} \approx 8 \mathrm{ft}^{2}$
b. white and blue both $18 \mathrm{ft}^{2}$
33. Area of orange center disc is $36 \pi$ in. $^{2} \approx 113$ in. ${ }^{2}$; area of green is $(64 \pi-36 \pi)$ in. $^{2} \approx 88$ in. ${ }^{2}$; area of purple is $(100 \pi-64 \pi)$ in. $^{2} \approx$ 113 in. ${ }^{2}$.
34. a. Both formulas make sense as approximations. Kaylee replaced radius by $\frac{d}{2}$ and $\pi$ by 3 ; Cassie simply replaced $\pi$ by 3.
b. See answer to part (a).
c. $\left(7.5^{2} \cdot \pi\right) \mathrm{cm}^{2} \approx 176.7 \mathrm{~cm}^{2}$
d. Radius $^{2}=\frac{98}{\pi}$ in. $^{2} \approx 31.2$ in. ${ }^{2}$; radius $=\sqrt{31.2} \mathrm{in} . \approx 5.6 \mathrm{in}$. Note: Students may not know about square roots. So, they will most likely solve this by guess and check. Step 1: radius square $=31.2$. Guess 5 , which is too low. Guess 6 , which is too high. Guess 5.5 , which is too low, etc.

Connections
35. a. The length of a belt is related to waist circumference.
b. The waist size of a pair of jeans is related to waist and hip circumference.
c. Hat size is related to head circumference.
d. Shirt size is related to neck, arm, and chest circumference.
36. a. The circumference of an 18-inch pizza would be $18 \pi$ in. $\approx 56.55$ in.; for a 21 -inch pizza, the circumference would be $21 \pi$ in. $\approx 65.97$ in.
b.

Pizza Measurements

c. The graph shows that circumference is a linear function of diameter with slope about 3 (actually $\pi$, of course). Students are likely to say for every 3 -inch increase in diameter, there is an increase of 9 inches in circumference.
d. Circumference of a 20 -inch pizza would be about 63 inches.
e. Diameter of a pizza with circumference 80 inches would be about 25 inches.
37. 21. This looks like calculation for area of a rectangle with dimensions 2 and 10.5.
38. 56.71625. This looks like calculation of area for a circle with radius 4.25.
39. 55.6625. This looks like calculation of area for a triangle with base 15.25 and height 7.3.
40. $\frac{72}{20}=\frac{18}{5}=3 \frac{3}{5}$. This looks like calculation for area of a rectangle with dimensions $1 \frac{3}{5}$ and $2 \frac{1}{4}$.
41. 36. This looks like perimeter for a rectangle that is 8 by 10 .
42. 23.55 . This looks like circumference of a circle with diameter $7 \frac{1}{2}$.
43. Jorge is correct. Pi is a number and can be a measurement for a side of a rectangle.
44. vertical slice: rectangle

45. horizontal slice: square

46. slanting slice: Different polygons can occur, depending on the angle of the slice. For example, a triangular face will occur if you cut a small pyramid off a corner of the prism.

47. a. (See Figure 1.)
b. The third trial is closest to $\frac{\pi}{4}$. If you take the ratio from the fourth column of the table and multiply this by 4 , you
will get an approximation for $\pi$. For Trial 1, you get about 3.104. For Trial 2 , you get about 3.128. For Trial 3, you get about 3.14. Trial 3 was the closest approximation to $\pi=3.14159265$.

## Extensions

48. His head will move $12 \pi$ feet $\approx 37.7$ feet farther than his feet. The justification without doing much arithmetic relies on the Distributive Property. The circumference of movement for his head is $(42,000,000+12) \pi$ feet $=(42,000,000 \pi+$ $12 \pi)$ feet, which is the circumference of Earth (on which his feet move) plus $12 \pi$ feet.
49. a. The new circumference is $(42,000,000 \pi+3)$ feet; so the new diameter is that number divided by $\pi$, or $\left(42,000,000+\frac{3}{\pi}\right) \approx(42,000,000.95$ feet. Since the rope is exactly the same distance away from Earth at all points, the extra $\frac{3}{\pi}$, or 0.95 , foot of diameter is equally distributed on both ends of Earth's diameter. Thus, the rope will hang about $\frac{\left(\frac{3}{\pi}\right)}{2}$ feet above Earth, or about 0.48 feet off Earth's surface.
b. This is really the same problem as part (a), so the rope will hang about 0.48 inches from the person's waist. $\frac{(\pi d+3)}{\pi}$ inches $=d+\frac{3}{\pi}$ inches; of course, this assumes that a person's waist is approximately a circle.
c. The calculations in parts (a) and (b) show that the effect is independent of the original diameter. Each manifestation of the phenomenon is a scale model of the other.
50. a. 100 feet of fencing will enclose the patio.
b. $A \approx(8.5 \times 8.5 \times \pi) \mathrm{ft}^{2}$, or $227.0 \mathrm{ft}^{2}$ of plastic. (The $1-\mathrm{ft}$ overhang added 2 ft total to the diameter.)
c. Tubing around the edge of the pool will be $15 \pi$ feet $\approx 47$ feet.
d. $\left(600-7.5^{2} \cdot \pi\right) \mathrm{ft}^{2} \approx 423.3 \mathrm{ft}^{2}$, which is the area of the pool subtracted from the area of the whole patio, giving you the area of the ground covered with tiles.

Figure 1

| Trial | Dots Inside <br> the Circle | Dots Inside <br> the Square | Ratio: $\frac{\text { Dots Inside Circle }}{\text { Dots Inside Square }}$ |
| :---: | :---: | :---: | :---: |
| 1 | 388 | 500 | $\frac{388}{500}=0.776$ |
| 2 | 352 | 450 | $\frac{352}{450} \approx 0.782$ |
| 3 | 373 | 475 | $\frac{373}{475} \approx 0.785$ |

