Applications

1. **a.** 12 **b.** 18.5 **c.** 28
2. **a.** All side areas are 24 in.^{2}.
   b. All perimeters are 6 in.
   c. triangular base area: 1.75 in.^{2}; square base area: 2.25 in.^{2}; hexagonal base area: 2.625 in.^{2}

   Note: The height of both the triangle and hexagon is actually $\sqrt{3}$, but, in the spirit of what students might measure in a classroom activity, 1.75 is used as an approximation. Students might need a hint to break the hexagon into six congruent triangles.
3. **a.** surface area: 97.5 cm^2 (This approximate answer uses 2.5 cm as the height of the triangular bases. The height is actually $1.5\sqrt{3}$, which gives a surface area of 97.79 cm^2.)
   b. volume: 37.5 cm^3 (As above, this is an approximation.)
4. **a.** surface area: 90 cm^2 (This is an approximation using base area of 15 cm^2.)
   b. volume: 60 cm^3 (This answer uses the same base area approximation as above.)
5. **a.** surface area: 44 cm^2 (This is an approximation using base area of 7 cm^2.)
   b. volume: 21 cm^3 (This answer uses the same base area approximation as above.)
6. **a.** First, you find the area of the base and double that. Then you calculate the area of the rectangle that wraps around the prism, which is the perimeter of the base times the height. Add those two calculations to get the surface area.
   b. It is the same general process. The only difference is that the methods for finding the area of the base for a rectangular and a nonrectangular prism are different.
7. **a.** Find the area of the base and then multiply that by the height of the prism.
   b. It is the same general process. The only difference is that the methods for finding the area of the base for a rectangular and a nonrectangular prism are different.
8. **a.** 120 cm^3 **b.** 185 cm^3 **c.** 280 cm^3
9. **a.** triangular prism volume: 7 in.^{3}
   b. square prism volume: 9 in.^{3}
   c. hexagonal prism volume: 10.5 in.^{3}
   d. These results are consistent with the pattern relating areas and volumes in the experiments of Problem 2.1, because only the dimensions of the piece of paper the students started with have changed. It shows that the principles involved are independent of specific numerical values.
10. **a.** Surface area $\approx$ $3(10)(15) + 2(0.5)(10)(8.7) \approx 537$ cm^2
    b. Volume $\approx (43.5)(15) \approx 652.5$ cm^3
11. **a.** Surface area = $2(5)(0.5)(4.0)(2.75) + 5(10)(4) = 255$ cm^2
    b. Volume = $10(27.5) = 275$ cm^3
12. Students will use a variety of approximation strategies to estimate the areas of the bases. At this point, we are not expecting formula-based calculations.

    pentagonal prism: approximately 7(8.5), or 59.5 in.^{3}

    triangular prism: approximately 6(8.5), or 51 in.^{3}

    hexagonal prism: approximately 8.5(8.5), or 72.25 in.^{3}

    square prism: approximately 8(8.5), or 68 in.^{3}
13. The slant cut gives two solids with five faces on each—two trapezoids (one not seen in the drawing), one rectangle, and two triangles (the original top or bottom and the triangle face created by the slice). Neither figure is a prism.

The horizontal cut gives two triangular prisms with five faces each—three rectangular vertical faces and congruent triangular tops and bottoms.

The vertical cut gives two prisms. One is a triangular prism with 5 faces (3 rectangles and 2 triangles). The other is a prism with six faces (trapezoidal base and top and 4 rectangular faces).

14. **a.** The first cut placement will yield two triangular pyramids, each with 4 vertices, 6 edges, and 4 triangular faces. The pyramids will be congruent if the cut coincides with two edges which run from the vertex of the pyramid to opposite vertices of the base.

b. The second cut placement will yield two figures. The shapes of those figures depend on how the cut is actually made. If the cut is parallel to the left face, then neither pyramids nor prisms are formed. The smaller will have two triangular faces (like the one shown), two trapezoid faces, and one rectangle face (on the bottom). The larger will have one triangular face (visible), two trapezoid faces like the one visible, a rectangle face (on the bottom), and another trapezoid face where the cut occurs. If the cut is made in a different way, it is possible to get one pyramid and one other figure that is neither pyramid nor prism.

Connections

15. **a.** The prism has 6 vertices, 9 edges, and 5 faces.

b. The greatest numbers you could see would be 6 vertices, 8 edges, and 3 faces. To see this much, you could position your eye above and away from the prism in such a way that you are facing directly toward a vertical edge. Other viewing positions are possible.

c. The smallest numbers you could see would be 3 vertices, 3 edges, and 1 face. To see this little, you could position yourself to look directly at the center of the top of the building.

16. **a.** The prism has 10 vertices, 15 edges, and 7 faces.

b. The greatest numbers you could see would be 9 vertices, 12 edges, and 4 faces, by looking from a location in front of and above one of the vertical faces (so you see the front face, two other lateral faces, and the top).

c. The smallest numbers you could see would be 4 vertices, 4 edges, and 1 face. To see this little, you could position yourself very close to and looking directly at the center of one of the vertical (rectangular) faces.

17. **a.** The pyramid has 5 vertices, 8 edges, and 5 faces.

b. The greatest numbers you could see would be 5 vertices, 8 edges, and 4 faces. To see this much, you could position your eye above the pyramid looking straight down on it.

c. The smallest numbers you could see would be 3 vertices, 3 edges, and one face. To see this little, you could position yourself very close to and looking directly at the center of one triangular face.

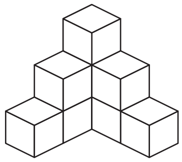
18. **a.** $\frac{35}{4} = 8\frac{3}{4}$. Each square in the grid is a 1×1-square unit. The area of the outlined rectangle is $3\frac{1}{2} \times 2\frac{1}{2}$. You can add the square units and parts of square units to find the total area, $8\frac{3}{4}$ square units.

b. 8.75 See answer to part (a) with the additional notation that $0.75 = \frac{3}{4}$ as a common fraction.

19. **a.** Yes. It is the sum of areas of eight triangles, each with base 3 and height $\frac{1}{2}(7.2)$.

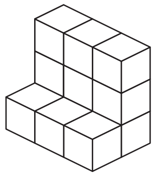
b. Yes. It is the area of the enclosing square minus the areas of the four corner triangles.

20. Sketches will vary. Possible sketch:



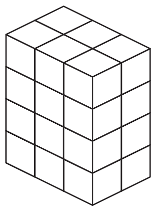
Volume: between 7 and 9 cubic units
(**Note:** There may be some hidden blocks.)

21. Sketches will vary. Possible sketch:



Volume: between 10 and 12 cubic units
(**Note:** There may be some hidden blocks.)

22. Sketches will vary. Possible sketch:



Volume: between 18 and 24 cubic units
(**Note:** There may be some hidden blocks.)

23. a. Volume increases by a factor of 2.25, or 1.5^2 (to 405 cm^3).

b. Volume increases by a factor of 9, or 3^2 (to 1,620 cm^3).

c. Volume reduces by a factor of 0.36, or 0.6^2 (to 64.8 cm^3).

d. Volume increases by a factor of 3 (to 540 cm^3).

e. Volume increases by a factor of $\frac{18}{15}$, or 1.2 (to 216 cm^3).

24. a. Rectangles A and C are similar because corresponding sides are related by the same scale factor, 2. You can also say that the ratio of length to width in both rectangles is 3 : 2. Rectangles B and D are similar because the scale factor of corresponding sides is 2.5, or because the length-to-width ratio is 2 : 1 in both figures.

b. The width of the fifth rectangle would have to be half the length in order to have the same length-to-width ratio as rectangles D and B. So, the width of the fifth rectangle would be 6.

25. Student answers might vary on what they can see standing inside a rectangular prism. If you assign this ACE exercise, warn students in advance that there is no exact correct answer—only answers that can be explained with some logic!

a. By standing or sitting in a corner, one could see all but two vertices and the one edge against which you are standing—so one could see 6 vertices, 11 edges, and 6 faces.

b. By standing facing into a corner, one can imagine being able to see only 2 vertices, 5 edges, and 4 faces; by standing right against one wall one could imagine being able to see 4 vertices, 4 edges, and 4 faces (depending on the capacity of one's peripheral vision).

26. a. $11\frac{1}{4}$ of the smaller container will fit in the larger container.

b. We use division $\left(3\frac{3}{4} \div \frac{1}{3}\right)$ because we want to know how many times a smaller number fits into a larger number.

27. a. $3\frac{3}{5}$ of the smaller container will fit in the larger container.

b. Use division $\left(2\frac{2}{5} \div \frac{2}{3}\right)$ because we want to know how many times a smaller number fits into a larger number.

28. a. 12; The tank's total height is 8 inches, but it is only filled to a height of $\frac{2}{3}$ inch. So, $8 \div \frac{2}{3} = 12$.

b. $\frac{1}{12}$

c. $768 \div 12\frac{3}{4} = 768 \div \frac{51}{4} = \frac{3{,}072}{51} = 60\frac{4}{17}$

29. a. $2\frac{1}{2}$; The tank's total height is 4 inches, but it is only filled to a height of $1\frac{3}{5}$ inches. So, $4 \div 1\frac{3}{5} = 2\frac{1}{2}$.

b. $1\frac{3}{5} \div 4 = \frac{2}{5}$

c. $300 \div 4\frac{4}{9} = 67\frac{1}{2}$

Extensions

30. **a.** One can multiply the base area times the height to get a volume of 40 cm^3.

b. 40 cm^3 is equal to the exact volume of the prism. Multiplying the area of the base and the height will give the exact volume of any prism, no matter how irregular its bases may be.

31. **a.** See Figure 1.

b. Euler's formula is expressed in a variety of algebraically equivalent ways. One is $E = V + F - 2$.

c. The formula holds for any solid with polygonal faces. For example, a pyramid with pentagonal base has 6 vertices, 10 edges, and 6 faces; and $10 = 6 + 6 - 2$.

Figure 1

Euler's Formula

Figure	Vertices	Edges	Faces
Rectangular Prism	8	12	6
Triangular Prism	6	9	5
Pentagonal Prism	10	15	7
Hexagonal Prism	12	18	8
Triangular Pyramid	4	6	4
Square Pyramid	5	8	5