Applications

1. Stack A: 64

   Stack B: 40

2. a. 25 pallets

   b. 60 pallets

   c. 180 pallets

3. a. Possible box dimensions are $1 \times 1 \times 40$; $1 \times 2 \times 20$; $1 \times 4 \times 10$; $1 \times 5 \times 8$; $2 \times 2 \times 10$; $2 \times 4 \times 5$.

   b. The box that is $2 \times 4 \times 5$ has surface area of 76 square inches, the smallest of all cases in part (a). This box has the least surface area because it is the most cube-like.

4. a. The box with dimensions closest to equal will have minimum surface area.

   b. The surface area of the $12 \times 12 \times 16$ box is 1,056 cm^2; the $12 \times 24 \times 8$ boxes have surface area 1,152 cm^2; the $4 \times 12 \times 48$ box has surface area 1,632 cm^2.

5. a. The volume is 525 cm^3; the surface area is 475 cm^2.

   b. The box of minimum surface area and volume 250 cm^3 would be a cube with edges approximately 6 cm $(\sqrt[3]{250} \approx 6.3)$ in length.

6. a. The surface area is 2,288 cm^2.

   b. The surface area is 2,128.75 cm^2.

7. a. The surface area is 3,243.5 cm^2.

   b. The surface area is 3,620.5 cm^2.

8. a. You can double the volume by doubling any one of the dimensions, but many other changes would have the same effect (e.g., multiplying one dimension by 4 and halving another dimension; multiplying each dimension by the cube root of 2 (not something we would expect from students at this level)).

   b. You can triple the volume by tripling any one of the dimensions, but many other changes would have the same effect.

   c. You could halve the volume by halving any one of the dimensions, but, again, many other changes would have the same effect.

   d. **i.** 3,072 in.^{3} **ii.** 1,296 in.^{3} **iii.** 48 in.^{3}

   You can find these answers by multiplying the original box volume by the factor f^3 in each case; that is, by multiplying by 8, 3.375, and 0.125.

9. a. No, your friend is incorrect. Doubling each dimension increases the volume by a factor of 8.

   b. No, your friend is incorrect. Doubling each dimension increases the cost of materials for the box by a factor of 4 (assuming that surface area is directly proportional to cost). It also increases the number and cost of worms by a factor of 8.

10. a. The landfill will hold 11,900,000 cubic feet.

    b. You need to know the size of the population and how much garbage, on average, each person produces in a given period of time. More directly, you need to know the total volume of garbage the city produces in a given period of time.

    c. Yes; $1.1^3 \approx 1.33$.

11. a. The actual ship will be 5,000 cm, or 50 meters long.

    b. $\frac{3{,}000 \text{ cm}}{200} = 15 \text{ cm}$

    c. $20 \text{ cm}^2 \times 200^2 = 800{,}000 \text{ cm}^2$, or 80 m^2

    d. $100 \text{ cm}^3 \times 200^3 = 800{,}000{,}000 \text{ cm}^3$, or 800 m^3

Connections

12. **a.** The only factor pair is (11, 1), and the prime factorization is $11 \cdot 1$.

b. The factor pairs are (18, 1), (9, 2), (6, 3), and the prime factorization is $2 \cdot 3 \cdot 3$.

c. The factor pairs are (42, 1), (21, 2), (14, 3), (7, 6), and the prime factorization is $2 \cdot 3 \cdot 7$.

13. **a.** Perimeter is 2.4 times 23.7, or 56.88, which is 2.4 times the perimeter of the original triangle.

b. Area is $2.4^2(0.5)(8.7)(5) = 125.28$, found by multiplying the area of the original triangle by the square of the scale factor.

c. 30°, 60°, and 90°; corresponding angles in the enlarged triangle will be equal in degree measure to the angle measures in the original triangle.

14. Area = 18 in.^{2} ; perimeter = 17.5 in.

15. Area = 30.59 cm^2; perimeter = 26 cm

16. Area = 22 cm^2; perimeter = 20 cm

17. 102°

18. B

19. A

20. $\frac{11}{12} \div \frac{1}{8} = 7\frac{1}{3}$. Ms. Zhou can make 7 doll beds and have enough left (if it is usable) for $\frac{1}{3}$ of another bed. (**Note:** Students might also realize that $1 - \frac{1}{12} > \frac{7}{8}$, so she can make 7 beds.)

21. **a.** $I = 12F$

b. $F = I \div 12$

c. $M = C \div 100$

d. $C = 100M$

e. $C = I \div 2.54$

22. **a.** 16 ft^2

b. The cost of 100 boxes will be $500 + 100(16)(\$.15) = \$740$.
The cost of 200 boxes will be \$980.
The cost of 1,000 boxes will be \$2,900.

c. $C = 500 + 2.40n$; since the equation is in the form $y = b + mx$, and since the relationship has a constant rate of change (2.40), the equation is linear. Note that the cost per square foot is \$.15 and that each box has a surface area of 16 ft^2, so the cost per box is \$.15 × 16, or \$2.40.

d. $750 = 500 + 2.40n$ when $n = 104.1666\ldots$; so the school can order at most 104 boxes.

23. $h = 4$ cm

24. $h = 3$ cm

25. **a.** $\frac{3}{20}$

b. Student diagrams will vary. A diagram could show a rectangle split into 5 horizontal bands of equal width and 3 of those bands shaded. Then the shaded section should be divided by three vertical lines to give 20 small pieces with three in each of four columns.

26. **a.** $\frac{7}{2} \div \frac{3}{8} = \frac{28}{3}$, so Mr. Bouck can make 9 full batches.

b. Student diagrams will vary. One possible diagram would show a long bar with 3 large segments and one half-segment marked; then each large segment would be divided into eight equal pieces and the half-segment into 4 equal pieces. Counting off by 3 (eighths), you would find 9 groups of $\frac{3}{8}$ and have one piece of length $\frac{1}{8}$ left over.

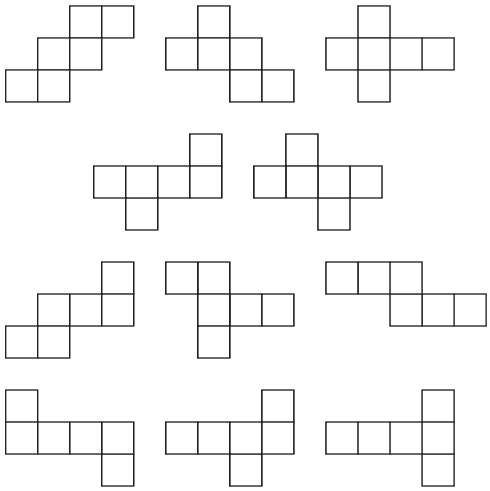
27. $4\frac{1}{3}$ full scoops should give 65% of the tank volume.

28. **a.** (150 ft + 90 ft)(20 ft)(2) = 9,600 square feet; 9,600 square feet ÷ 400 square feet/gallon = 24 gallons of paint

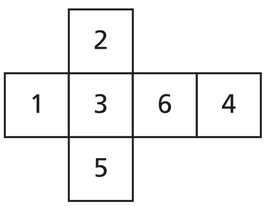
b. 24(\$25.50) = \$612.00

Extensions

29. a. There are many possible nets for a cube.



b. There are many ways to arrange the numbers on a net. Here is one possible arrangement of the numbers on one possible net.



c. On the plan shown, you could swap the positions of the 6 and 1. You could exchange the positions of the 6 and 1 with those of the 2 and 5. The only condition is that the same sum-of-7 pairs have to be in positions where they will be opposite when you fold the net.

30. a. Each net has area of 6 in.^{2}, so area conditions would suggest exactly 6 nets from a 9 inch $\times$ 4 inch rectangle.

b. You cannot use the given net for a cube to exactly cover a rectangle because there is no way to cover the corner squares of a rectangular grid with the given shape (unless some of the net spills over the edge of the rectangle).

31. a. 260.7. This is the surface area, in square centimeters, of Box C.

b. $37\frac{1}{2}$. This is the surface area, in square centimeters, of Box B. (The 6 corresponds to the number of faces on the cube; the $6\frac{1}{2}$ corresponds to the area of each face.)

c. 39. This is the volume, in cubic centimeters, of Box A. (The 6 corresponds to the area of the base; the $6\frac{1}{2}$ corresponds to the height of the prism.)

d. 28. This is the volume, in cubic centimeters, of Box D.

32. a. There are several possible formulas. One is S.A. $= 2(\ell w + \ell h + wh)$.

b. After scaling, the formula suggests this surface area: S.A. $= 2(f\ell fw + f\ell fh + fwfh)$.

c. Factoring out two factors of f from each term in parentheses, you get the formula

S.A. $= 2f^2(\ell w + \ell h + wh)$, which is f^2 times the original area.

33. a. $V = \ell wh$

b. $V = (f\ell)(fw)(fh)$

c. You can rearrange the factors on the right side of the equation in part (b) to get $V = (fff)(\ell wh)$, which is equivalent to $f^3(\ell wh)$.