ACE Answers | Investigation 4

Applications

1. Answers will vary. Possible answers: Yes, because there is only one blue marble in Bucket 1 and only one red marble in Bucket 2. Yes, because it is going to be very hard to choose both a red and a blue.

2.

	Red	Yellow	Green
Red	R,R	R,Y	R,G
Blue	B,R	B,Y	B,G
Green	G,R	G,Y	G,G
Yellow	Y,R	Y,Y	Y,G

Each marble in Bucket 1 is matched to each marble in Bucket 2. There are 12 outcomes.

3. Since only one outcome has a blue-red, the probability is $\frac{1}{12}$.

4. **a.** $\frac{1}{12}$

   b. $\frac{9}{12} = \frac{3}{4}$

   c. $\frac{0}{12} = 0$

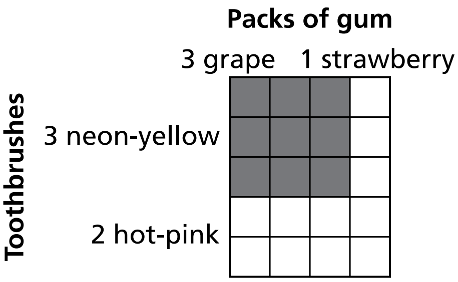
   d. $\frac{3}{12} = \frac{1}{4}$

5. **a.**

	Red	White	Blue	Green
Red			▨	
White				
Blue	▨			

   b. $\frac{2}{12} = \frac{1}{6}$

6. **a.**



Kira has a $\frac{9}{20}$ probability of drawing a neon-yellow toothbrush and a pack of grape gum.

   b. Of 100 patients, you could expect about 45 $\left(\frac{9}{20} \times 100\right)$ to draw the same prizes Kira chose.

7. Deion should score 7 (or 14) points when the spinners make purple and Bonita should score 2 (or 4, if Deion scores 14) points when the spinners do not make purple. This is because Deion has a $\frac{4}{18} = \frac{2}{9}$ chance of making purple and Bonita has a $\frac{14}{18} = \frac{7}{9}$ chance of not making purple. For example, if the spinners were spun 54 times, Deion would get purple 12 times, scoring 12 × 7 points in the long run, which equals 84 points; Bonita would not get purple 42 times, scoring 42 × 2 points in the long run, which equals 84 points.

8. **a.** There are nine possible outcomes: yellow-blue, yellow-orange, yellow-yellow, red-blue, red-orange, red-yellow, green-blue, green-orange, and green-yellow. These outcomes are equally likely, since each spinner has the same number of spaces and all the spaces are the same size.

   b. $\frac{1}{9}$; purple (red-blue) is one of the nine equally likely outcomes.

   c. About 11 people, since $9 \times 11 = 99$, and for every 9 people you would expect one winner.

   d. About \$45; if 100 people play, \$100 is collected. Since there are about 11 winners, $11 \times 5 = \$55$. $100 - 55 = \$45$.

9. **a.** Any of the methods could be used, though some might be easier to use than others.

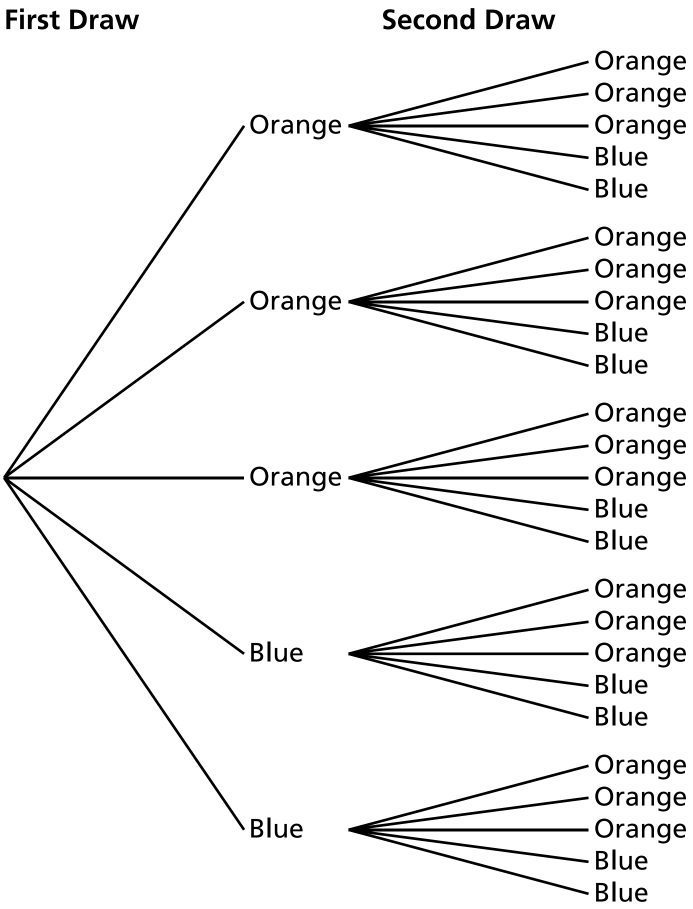
b. To use a tree diagram, five branches would represent the first draw. From each of these, another five branches would represent the second draw. This would result in 25 different outcomes. (See Figure 1.)

To use a list, you have to be careful to distinguish between the different orange and blue marbles (for example, by using symbols such as O1, O2, O3, B1, B2). You would list O1 with each of the other possibilities, and then O2 with each of the other possibilities, and so on. Again, you will find 25 possible outcomes.

To use an area model, you could divide a square into five columns to represent the first draw. Then, divide each column into five parts to represent the second draw. This will yield 25 subdivisions, as shown below.

		First Draw				
		O1	O2	O3	B1	B2
Second Draw	O1					
	O2					
	O3					
	B1					
	B2					

Figure 1



A chart would be similar to the area model, with five rows and five columns, as shown here:

	O	O	O	B	B
O	OO	OO	OO	OB	OB
O	OO	OO	OO	OB	OB
O	OO	OO	OO	OB	OB
B	BO	BO	BO	BB	BB
B	BO	BO	BO	BB	BB

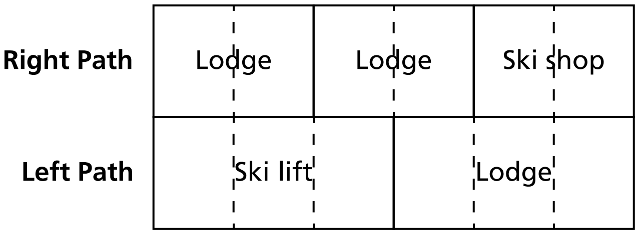
10. Of the 25 possible outcomes, 13 represent marbles of the same color, so you could expect to draw two marbles of the same color $\frac{13}{25}$, or 26 out of 50, times.

11. Possible answer: Award 12 points for a match, 13 points for a no-match.

12. a. Possible answer: At each fork that splits into two trails, if even is rolled, go to the right, and if odd is rolled, go to the left. At the fork that splits into three trails, if you roll a 1 or 2, choose the leftmost path; a 3 or 4, choose the middle path; and a 5 or 6, choose the rightmost path.

b. Answers will vary. Students should get probabilities similar to the following: P(lodge) = $\frac{7}{12}$, P(lift) = $\frac{1}{4}$, P(ski shop) = $\frac{1}{6}$.

c.



For large numbers of experiments, the probabilities should be close to the theoretical probability of $\frac{7}{12}$ for the lodge, $\frac{1}{4}$ for the lift, $\frac{1}{6}$ for the ski shop.

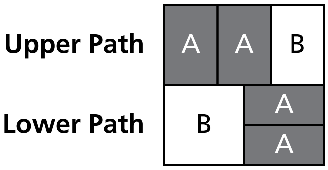
13. a. Cave A: $\frac{7}{12}$

$\frac{1}{6}+\frac{1}{6}+\frac{1}{8}+\frac{1}{8}=\frac{7}{12}\approx 58\%$

Cave B: $\frac{5}{12}$

$\frac{1}{6}+\frac{1}{4}=\frac{5}{12}\approx 42\%$

A square can be divided to show the probability of a player ending in each cave, as shown below.



b. If you played the game 100 times, you could expect to end in Cave A $100 \times \frac{7}{12} \approx 58$ times and in Cave B $100 \times \frac{5}{12} \approx 42$ times.

14. Cave A:

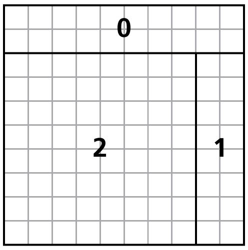
$\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{4}=\frac{3}{4}$

Cave B:

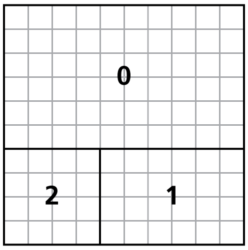
$\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$

15. B

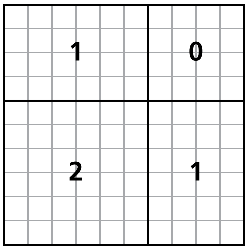
16. As shown in the area model below, this player is most likely to score 2 points.



17. As shown in the area model below, this player is most likely to score 0 points.



18. **a.** As shown in the area model below, Nishi is most likely to score 1 point.



b. In 100 attempts, Nishi could expect to score 2 points 36 times, 1 point 48 times, and 0 points 16 times, for an overall total of 36(2) + 48(1) + 16(0) = 120 points. The average number of points she could expect per trip is thus 120 ÷ 100 = 1.2.

19. **a.** A player with a 50% free-throw percentage would be most likely to score 1 point, getting 1 point per situation.

b. A player with an 80% free-throw percentage would be most likely to get 2 points, getting 1.6 points per situation.

20. **a.** David has the best chance of making his next free throw, as his percent of free throws made is the highest.

b. Gerrit 50%, David 79.6%, Ken 61.6%, Alex 70%

21. **a.** P(0 points) = 30%, P(1 point) = 21%, P(2 points) = 49%

b. 0 points: 30 times; 1 point: 21 times; and 2 points: 49 times.

c. The average number of points he could expect per trip is 119 ÷ 100 = 1.19.

d. Students may want to use 10 by 10 grids. P(0 points) = 50%, P(1 point) = 25%, P(2 points) = 25%.

22. **a.** Students may want to use a tree diagram. P(0 points) = 25%, P(1 point) = 50%, and P(2 points) = 25%.

b. The probabilities for making both free throws in the two situations are equal. The other two probabilities are different because in the two-free-throw situation, Gerrit can score 1 point by either making the first free throw and missing the second, or missing the first free throw and making the second. The fact that he automatically gets a second free throw increases his probability of getting 1 point and reduces his probability of getting 0 points.

Connections

23. $\frac{15}{100} = \frac{3}{20}$

24. $\frac{6}{100} = \frac{3}{50}$

25. $\frac{28}{100} = \frac{7}{25}$

26. $\frac{30}{100} = \frac{3}{10}$

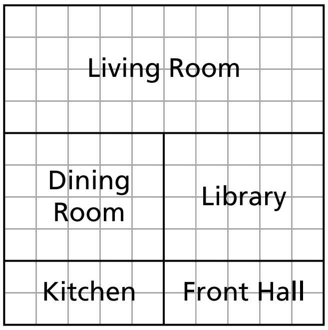
27. $\frac{21}{100}$

28. F

29. Drawings will vary. Below is a sample floor plan. According to the data, Carlos played the game 50 times, and the experimental probabilities that the treasure will be in each room are as follows:

$$P(\text{dining room}) = \frac{10}{50} = 0.20$$
$$P(\text{living room}) = \frac{20}{50} = 0.40$$
$$P(\text{library}) = \frac{10}{50} = 0.20$$
$$P(\text{kitchen}) = \frac{5}{50} = 0.10$$
$$P(\text{front hall}) = \frac{5}{50} = 0.10$$

On a 10-by-10 grid, the dining room should occupy 20 squares, the living room 40 squares, the library 20 squares, the kitchen 10 squares, and the front hall 10 squares.



30. **a.** There are 60 seniors that drive.

b. There are 70 seniors who favor the rule.

c. There are 30 seniors who favor the rule and do not drive to school.

d. 70 of the 100 seniors surveyed favor the rule; the probability is $\frac{70}{100} = \frac{7}{10}$, or 70%.

e. 40 of the 100 seniors surveyed drive to school and favor the rule; the probability is $\frac{40}{100} = \frac{2}{5}$, or 40%.

f. Of the 100 seniors surveyed, 60 drive to school. Of those who don't drive to school, 10 oppose the rule. This is a total of 70, so the probability is $\frac{70}{100} = \frac{7}{10}$. **Note:** Adding the total number of seniors who drive to school (60) to the total number who oppose the rule (30) is incorrect because it double-counts those who drive to school and oppose the rule. Another way to do this problem is to determine who is not counted, seniors who do not drive to school and who favor the rule, for a total of 30, which leaves 70.

g. One problem with this survey is that it polled only seniors. Since the question concerns a rule that would allow only seniors to drive, many of the other students at the school might oppose it. Students old enough to drive in other grade levels might be more likely to oppose the rule. Thus this survey is probably not a good indicator of the opinions of the entire student body.

31. **a.**

First spin

Second spin		−1	2	3	−4
	−1	−2	1	2	−5
	2	1	4	5	−2
	3	2	5	6	−1
	−4	−5	−2	−1	−8

b. Yes, Marni and Ira are equally likely to win. As shown above, positive and negative sums are equally likely $\left(\frac{8}{16}\right)$.

32. **a.** $\frac{20}{36} = \frac{5}{9}$ **b.** $\frac{16}{36} = \frac{4}{9}$

33. **a.** $P(\text{landing on A for Dartboard 1}) = \frac{18}{36} = \frac{1}{2}$;

$P(\text{landing on B for Dartboard 1}) = \frac{18}{36} = \frac{1}{2}$.

$P(\text{landing on A for Dartboard 2}) = \frac{19}{36}$;

$P(\text{landing on B for Dartboard 2}) = \frac{17}{36}$.

Each dartboard can be divided up into 36 equal pieces that are the size of the smallest square piece on each board.

b. For each dartboard, a person pays \$36 to play 36 times.

For Dartboard 1: A person would make \$0 since the probability of landing on B is $\frac{18}{36}$, or 18 out of 36 times and the payment is \$2(18) = \$36, which is the same as the cost to play.

For Dartboard 2: A person would lose \$2 since the probability of landing on B is $\frac{17}{36}$, or 17 out of 36 times and the payment is \$2(17) = \$34, which is \$2 less than the cost to play.

c. The carnival would make \$0 from Dartboard 1 and \$2 from Dartboard 2.

d. Assuming the darts land at random, the carnival can expect to make a profit of \$2 for each 36 plays or 5.6 cents per play on this game if players choose Dartboard 2. The carnival can expect to break even on Dartboard 1.

34. a. The factors of 5 are 5 and 1, so there is a 2 out of 6 or $\frac{1}{3}$ chance on each roll of getting a factor of 5. The probability of getting a factor of 5 on two consecutive rolls is $\frac{1}{9}$. **Note:** Students may list all possible combinations, draw an area model, or, if they are beginning to see the connection to multiplying fractions, compute $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

b. $\frac{4}{36}$, or $\frac{1}{9}$

c. The answers are the same; rolling the same number cube a second time is equivalent to rolling a second number cube. Each roll of a number cube is independent of other rolls.

35. Each section is equally likely, so there is a 1 in 10, or 10%, chance of landing on bankrupt.

36. Three of the 10 sections have a value of \$500 or more, so the probability of getting at least \$500 on one spin is $\frac{3}{10}$, or 30%.

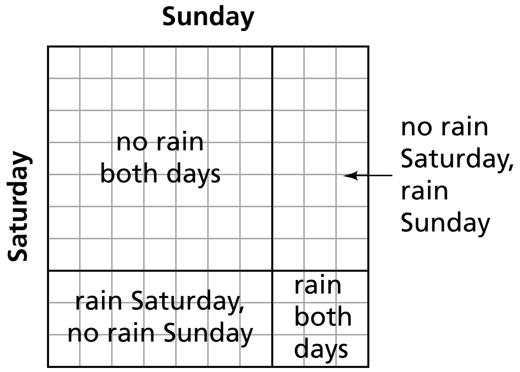
37. It is still 10%. Each player has a 10% chance of hitting \$350 on each spin.

38. B

39. F

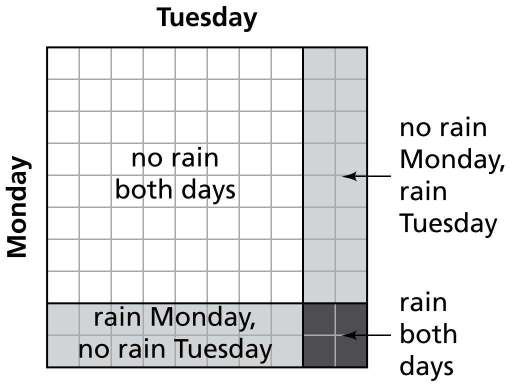
40. C

41. a. As shown in the area model below, the probability that it would rain on both days is 9%. **Note:** After working with many problems such as this, students may begin to multiply probabilities. The probability that it would rain on both days is $\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$, or 9%.



b. Wanda's prediction stated that there was a 9% chance that it would rain on both days. She should explain to her manager that while it was unlikely that it would rain both days, it was not impossible. Wanda's knowledge should not be in question just because she predicted something that had only a 9% chance of happening.

c. As shown in the area model below, there is a 36% chance that it will rain on at least one of the two days.



42. 3,000 salmon, since $\frac{150}{500} = \frac{3{,}000}{10{,}000}$

43. **a.** (See Figure 2.)

b. No. The average for a player with an 80% percentage is more than twice that of a player with a 40% percentage.

c. Either from the table or the graph we can see that the average for a player with a 50% percentage has to be between 0.56 and 0.96. This is not half the average for a player with a 100% percentage.

d. This table shows that the rate stays the same, no matter how many free throws are taken. So if the player takes 50 free throws, the average made will be half the average if the player takes 100 free throws. (See Figure 3.)

44.

	4	·3	·1
4	8	1	3
·3	1	−6	−4
·1	3	−4	−2

a. $\frac{5}{9}$

b. $8 + 1 + 3 + 1 - 6 - 4 + 3 - 4 - 2 = 0$; $\frac{0}{9} = 0$

45.

	4	·3	·1
4	16	−12	−4
·3	−12	9	3
·1	−4	3	1

a. $\frac{5}{9}$

b. $16 - 12 - 4 - 12 + 9 + 3 - 4 + 3 + 1 = 0$; $\frac{0}{9} = 0$

Figure 2

Probability of One Basket	20%	40%	60%	80%	100%
Average Points per One-and-One Attempt	0.24	0.56	0.96	1.44	2.0

Figure 3

Number of One-and-One Situations by a Player With a 20% Pecentage	01	1	20	100
Average Points Made	0.24	2.4	4.8	24

Extensions

46. **a.** See first two columns in table below.

b. See last column in table below. The containers can be reversed, but the probabilities for the arrangements remain the same.

Container 1	Container 2	*P*(Green)
BBGG	empty	$\frac{1}{4}$
BBG	G	$\frac{2}{3}$
BB	GG	$\frac{1}{2}$
B	GGB	$\frac{1}{3}$
BG	BG	$\frac{1}{2}$

Note the following shows how to find each probability:

P(Green) for BBGG $= \frac{1}{2} \times \frac{2}{4} = \frac{2}{8} = \frac{1}{4}$;

P(Green) for BBG =
$\left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{2} \times \frac{1}{1}\right) = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$

P(Green) for BB $= \frac{1}{2} \times \frac{2}{2} = \frac{2}{4} = \frac{1}{2}$;

P(Green) for B $= \frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$,

P(Green) for BG =
$\left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

c. The arrangement of one green marble in one container and the three other marbles in the second container gives them the best chance of winning. The probability of drawing green from the arrangement of one green marble in one container and the three other marbles in the second container is $\frac{2}{3}$. When you use a square to analyze this option, the area for green is greater than in the other options. The arrangement of all four marbles in one container gives them the worst chance of winning. The probability of drawing a green marble with this arrangement is only $\frac{1}{4}$.

47. Note that the probabilities are given here for reference. The containers can be reversed, but the probabilities for the arrangements remain the same. (See Figure 4.)

The best arrangement is one green marble in one container and the remaining marbles in the other container. The probability of choosing green is $\frac{3}{4}$.

Figure 4

Container 1	Container 2	*P*(Green)	Work
BBGGG	empty	$\frac{3}{10}$	$\frac{1}{2} \times \frac{3}{5}$
BBGG	G	$\frac{3}{4}$	$\left(\frac{1}{2} \times \frac{2}{4}\right) + \left(\frac{1}{2} \times 1\right) = \frac{1}{4} + \frac{1}{2}$
BBG	GG	$\frac{2}{3}$	$\left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{2} \times 1\right) = \frac{1}{6} + \frac{1}{2}$
BB	GGG	$\frac{1}{2}$	$\frac{1}{2} \times 1$
B	GGGB	$\frac{3}{8}$	$\frac{1}{2} \times \frac{3}{4}$
BG	GGB	$\frac{7}{12}$	$\left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{2}{3}\right) = \frac{1}{4} + \frac{1}{3}$

48. Note that the probabilities are given here for reference. The containers can be reversed, but the probabilities for the arrangements remain the same. (See Figure 5.)

The best arrangement is to put one green marble in the first container, one green in the second container, and the two blue marbles in the other container. The probability of choosing green is $\frac{2}{3}$.

49. Note that the probabilities are given here for reference. The containers can be reversed, but the probabilities for the arrangements remain the same. Since we are concerned about the red marbles, the yellow and green marbles will be grouped together and labeled O, for other. (See Figure 6.)

Della should put one red marble in one can and the remaining marbles in the other can. This will give her a $\frac{3}{5}$ probability of winning.

Figure 5

Container 1	Container 2	Container 3	*P*(Green)	Work
GGBB	empty	empty	$\frac{1}{6}$	$\frac{1}{3} \times \frac{2}{4} = \frac{2}{12}$
GGB	B	empty	$\frac{2}{9}$	$\frac{1}{3} \times \frac{2}{3}$
GG	BB	empty	$\frac{1}{3}$	$\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$
GG	B	B	$\frac{1}{3}$	$\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$
G	G	BB	$\frac{2}{3}$	$\left(\frac{1}{3} \times 1\right) + \left(\frac{1}{3} \times 1\right) = \frac{1}{3} + \frac{1}{3}$
G	BBG	empty	$\frac{4}{9}$	$\left(\frac{1}{3} \times 1\right) + \left(\frac{1}{3} \times \frac{1}{3}\right) = \frac{1}{3} + \frac{1}{9}$
G	BG	B	$\frac{1}{2}$	$\left(\frac{1}{3} \times 1\right) + \left(\frac{1}{3} \times \frac{1}{2}\right) = \frac{1}{3} + \frac{1}{6}$
GB	GB	empty	$\frac{1}{3}$	$\left(\frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

Figure 6

Container 1	Container 2	P(Red)	Work
RROOOO	empty	$\frac{1}{6}$	$\frac{1}{2} \times \frac{2}{6}$
RROOO	O	$\frac{1}{5}$	$\frac{1}{2} \times \frac{2}{5}$
RROO	OO	$\frac{1}{4}$	$\frac{1}{2} \times \frac{2}{4}$
RRO	OOO	$\frac{1}{3}$	$\frac{1}{2} \times \frac{2}{3}$
RR	OOOO	$\frac{1}{2}$	$\frac{1}{2} \times 1$
R	ROOOO	$\frac{3}{5}$	$\left(\frac{1}{2} \times 1\right) + \left(\frac{1}{2} \times \frac{1}{5}\right) = \frac{1}{2} + \frac{1}{10} = \frac{6}{10}$
RO	ROOO	$\frac{3}{8}$	$\left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{4}\right) = \frac{1}{4} + \frac{1}{8}$
ROO	ROO	$\frac{1}{3}$	$\left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

50. Answers will vary.

51. One way to solve this is to make tables, as below. Within each table, each cell represents an equally likely outcome.

		Spinner A			
		Green	Blue	Red	Yellow
Spinner A	Green				
	Blue			▨	
	Red		▨		
	Yellow				

		Spinner B		
		Green	Red	Blue
Spinner B	Green			
	Red			▨
	Blue		▨	

		Spinner B		
		Green	Red	Blue
Spinner A	Green			
	Blue		▨	
	Yellow			
	Red			▨

The greatest chance of winning is in spinning spinner B twice. Possible explanation: The charts show that each way to spin the spinners results in two red-blue pairs; the way with the fewest possible outcomes is the best choice.

52. Approximately 34%; the probability that Alex will make one free throw is 70%, or $\frac{7}{10}$. Out of 100 sets of three free throws, he will make the first free throw about 70% of the time, or 70 times. Out of those 70 times, he will make the second free throw about 70% of the time, or $0.7 \times 70 =$ 49 times. Out of those 49 times, he will make the third free throw about 70% of the time, or $0.7 \times 49 =$ about 34 times. Thus, the probability that he will make his next three free throws is about 34%.

53. Approximately 64%; David's free-throw percentage is 79.6%, or about 80%. Out of 100 attempts, David can expect to make 80 of his first free throws. Of his 80 second free throws, he can expect to make the second free throw 80% of the time, or about 64 times. This gives him about a 64% chance of making both free throws. **Note:** Students might also multiply probabilities: $\frac{39}{49} \times \frac{39}{49} = \frac{1{,}521}{2{,}401}$, or about 63.3% of the time.

54. He should choose the option that allows him to make 4 out of 5. The probability of Emilio getting 3 in a row is $\frac{1}{8}$. However, there are 32 equally likely outcomes for 5 free throws. Of these 32 outcomes, 6 are successful (hitting all five, missing the first free throw and hitting the rest, missing the second free throw and hitting the rest, etc.) Therefore, the probability that he will get 4 out of 5 is $\frac{6}{32} > \frac{1}{8}$.

55. **a.** The probability that Curt will make three free throws in a row is $0.6 \times 0.6 \times 0.6 = 0.216$, or 21.6%.

b. Approximately 48%; there are four ways Curt could make three free throws and miss one. The probability that Curt will miss any one free throw is 40% or 0.4, so the probability that he will miss any one specific free throw and make the other three is $0.4 \times 0.6 \times 0.6 \times 0.6 = 0.0864$, or 8.64%. For all four ways he could miss one free throw. This is a combined probability of $4 \times 8.64\% = 34.56\%$. Curt could also make all four free throws, with a probability of $0.6 \times 0.6 \times 0.6 \times 0.6 = 0.1296$, or 12.96%. The probability, then, that he will make at least three out of four free throws is $34.56\% + 12.96\% = 47.52\%$.

56. **a.** 3 free throws: a

2 free throws: b, c, and e

1 free throw: d, f, and g

0 free throws: h

b. $P(1 \text{ point}) = 0.096$ $P(2 \text{ points}) = 0.384$

$P(3 \text{ points}) = 0.512$ $P(0 \text{ points}) = 0.008$