2.1 Sharing Pizza: Comparing and Scaling Rates

> *Focus Question* How can you determine whether two ratios are equivalent or find which of two ratios is more favorable?

Launch

Ask the following question, but do not analyze students' responses until the Summarize.

- *At which table would you choose to sit?*

Do not correct any of their ideas before the Explore phase.

Talk about what the Problem asks the students to do. Encourage groups to try to find more than one way to solve the Problem.

Explore

As you circulate, ask the following questions.

- *Can you think of another strategy to solve Question A? Are your two strategies related? If so, how are the strategies related?*

Question B requires students to analyze the reasoning of two other students and decide whether their methods are appropriate comparisons. In Question C, students set up equivalent ratios and solve for an unknown value.

Note: Not all students will use proportions to answer these Questions. Have students who use other methods share their strategies as well.

> *Key Vocabulary*
>
> There are no new glossary terms introduced in this Problem.
>
> *Materials*
>
> There are no additional materials for this Problem.

Summarize

Students will probably come up with a variety of ways to reason about this. Some may say that they would sit at the large table because it has more pizza. Others might choose the small table since it has fewer people. In either case, using only one number does not give enough information to choose wisely.

When students use certain strategies, such as scaling a ratio to an equivalent ratio, explicitly call attention to that strategy for the class. Some students will compute unit rates, which will show up in Problem 2.2.

> **Ⓐ Ⓒ Ⓔ**
> *Assignment Guide for Problem 2.1*
>
> Applications: 1–3 | Connections: 14–17
> Extensions: 28

Answers to Problem 2.1

A. No. Possible explanation: 4 pizzas for every 10 people (4 : 10) is equivalent to 16 pizzas for every 40 people (16 : 40), and 3 pizzas for every 8 people (3 : 8) is equivalent to 15 pizzas for every 40 people (15 : 40). Therefore, a person sitting at the large table gets more pizza since 16 pizzas for 40 people is greater than 15 pizzas for 40 people.

Note: Students might make equivalent ratios with the same number of pizzas, for example, 12 pizzas : 30 people and 12 pizzas : 32 people. If more people share the same number of pizzas, each person gets a smaller share.

Students might compare the amount of pizza per person or the number of people per pizza for the two tables. At the large table, each person will get $\frac{4}{10}$, or $\frac{2}{5}$, of a pizza. Those at the small table will get $\frac{3}{8}$ of a pizza per person. At the large table, there are $2\frac{1}{2}$ people per pizza. At the small table, there are $2\frac{2}{3}$ people per pizza. Therefore, each person at the large table will get more pizza than each person at the small table.

B. 1. No, Selena is incorrect. Even though there are six more people than pizzas at the large table and only five more people than pizzas at the small table, each person at the large table will get a greater share of pizza than each person at the small table.

Note: Some students will agree with Selena. For them, the comparison implied by a ratio is defined by the *difference* between the ratio parts. They think about how many extra people there are compared to the number of pizzas (six at the large table and five at the small table). The next part of this Question directly addresses this misconception.

2. Tony is correct. Placing 1 more pizza on the large table gives a ratio of 5 pizzas to 10 people, or $\frac{1}{2}$ pizza to 1 person. At the smaller table, sharing 3 pizzas among 8 people gives each person less than $\frac{1}{2}$ pizza. Selena's focus on differences leads to the conclusion that 5 : 10 and 3 : 8 result in each person at either table getting the same amount of pizza.

C. Students often use either proportions or scaling strategies to answer Question C.

1. 60 pizzas (20 tables); 3 pizzas : 8 people is equivalent to *x* pizzas : 160 people. The scale factor is 20, so we need $3 \times 20 = 60$ pizzas.

2. 64 pizzas (16 tables); 4 pizzas : 10 people is equivalent to *x* pizzas : 160 people. The scale factor is 16, so we need $4 \times 16 = 64$ pizzas.

3. Either 9 or 10 pizzas; The camp director would probably try to order a ratio of pizzas to people that is either equivalent to 3 : 8 or 4 : 10, or somewhere in between the two ratios.

4 : 10, scaled up by a factor of 2.5, is equivalent to 10 pizzas : 25 people. 3 : 8, scaled up by a factor of $\frac{25}{8}$ or 3.125, is equivalent to 9.375 pizzas : 25 people. Therefore, 10 pizzas should be on every extra-large table.

Note: Students might also solve this problem by finding the unit rate of pizza per person. The small table provides $\frac{3}{8}$ of a pizza per person; the large table provides $\frac{4}{10}$ of a pizza per person. The extra large table should provide a fraction of a pizza per person that is close to these fractions. Often, students find a fraction between these two fractions. A common fraction students find between $\frac{30}{80}$ and $\frac{32}{80}$ is $\frac{31}{80}$. If every person gets $\frac{31}{80}$ of a pizza, the camp director would need $25 \times \frac{31}{80} = 9.6875$ pizzas for every 25 people.

4. Answers may vary. Possible answers and explanations are below.

Depending on whether students choose to provide 9 or 10 pizzas per 25 people, the proportion is either $\frac{9}{25} = \frac{x}{160}$ (or scaling the ratio 9 : 25 by a factor of 6.4), or $\frac{10}{25} = \frac{x}{160}$ (or scaling the ratio 10 : 25 by a factor of 6.4).

Part (4) is complex because 160 people cannot be seated evenly at full 25-person tables. The scale factor, 6.4, is the number of 25-person tables needed to seat 160 people. Students may ignore this and find that the director needs to buy 9 or 10 pizzas for each of 6.4 tables. This would result in 58 pizzas for 9 pizzas per table, or 64 pizzas for 10 pizzas per table.

Alternatively, students may say that there will be 6 full tables (150 people), with 9 or 10 pizzas each, and one partial table (10 people) with 4 pizzas. This would result in 58 pizzas for 9 pizzas per extra-large table or 64 pizzas for 10 pizzas per extra-large table.