

Applications

1. a. 560
b. 78%
c. 39 to 11 (or 780 to 220)
2. a. $\frac{750}{2,000}$ or $\frac{3}{8}$
b. 62.5%; Here students need to recognize that the fraction they need is $\frac{5}{8}$, and $5 \div 8 = 0.625$.
c. 5 to 3 (or 1,250 to 750)
3. a. $\frac{5}{7}$
b. 60 people
c. about 71% (71.429%)
4. Possible answer: Fractions are a logical way to compare how students spent their time as they compare the time devoted to each activity (part) to the whole time investigated (whole).
5. a. No, $\frac{6}{48} = \frac{1}{8}$.
b. Yes, $6 : 2 = 3 : 1$.
c. Yes, $8 + 2 + 6 = 16$ and $\frac{16}{48} \approx 0.333$, or 33%.
d. No, $\frac{2}{6} \approx 0.333$, $0.333 \approx 33\% \neq 20\%$.
e. Yes, $18 + 2.5 + 9.5 = 30$; $48 - 30 = 18$; $30 - 18 = 12$.
6. Answers may vary.
7. Possible answers:
 - a. The ratio of hours Carlos spent sleeping to hours he spent watching television is 3 to 1. The ratio of hours spent on the phone to doing chores or homework is 1 to 1.
 - b. The difference between the number of hours Carlos spent sleeping and the number spent watching television is 2.
 - c. Carlos spent $\frac{8}{48}$ or $\frac{1}{6}$ of his time on recreation.
 - d. Carlos spent 50% of his time watching television and sleeping. Carlos spent about 33% of his time on recreation, watching television, and doing chores and homework.
8. a. 7 to 10
b. $\frac{(75 + 90)}{(180 + 240)} = \frac{11}{8}$
c. $\frac{75}{180} = \frac{5}{12}$
d. $\frac{225}{420} = \frac{17}{28} = 0.607$ or 61%
e. $\frac{90}{240} = 0.375$ or 37.5%
f. Grade 7; Grade 7 is 41.7% and Grade 8 is 37.5%.
9. Possible answers:
 - a. Students prefer radio to television by a ratio of 2 to 3 (2 : 3).
Students prefer television to radio by a ratio of 3 to 2 (3 : 2).
 - b. 60% of students prefer television and 40% of students prefer radio.
 - c. $\frac{3}{5}$ of students prefer television to radio.
 - d. The difference between the number of students who prefer television to radio is 20.
10. a. Mix Y is the most appley given it has the highest concentrate-to-juice ratio. The ratios of concentrate to juice are the following: Mix W = 5 : 13, Mix X = 3 : 9, Mix Y = 6 : 15, and Mix Z = 3 : 8.
b. Mix W = $\frac{5}{13}$, Mix X = $\frac{3}{9} = \frac{1}{3}$,
Mix Y = $\frac{6}{15} = \frac{2}{5}$, Mix Z = $\frac{3}{8}$
c. Mix W $\approx 38.5\%$, Mix X $\approx 33.3\%$,
Mix Y = 40%, Mix Z = 37.5%
d. Mix W: $\frac{8}{13}$ cup water and $\frac{5}{13}$ cup concentrate

- 11.** Examine these statements about the apple juice mixes in Exercise 1. Decide whether each is accurate. Give reasons for your answers.
- a.** Not accurate since both water and concentrate contribute to the least apple taste. A mix with 9 cups of water that had 1 cup of concentrate would taste much less apple.
 - b.** Not accurate. Mix Y is the most apple. Also, being the most apple is not dependent on the difference between the two ingredients, but on the fraction or percent of concentrate of the total cups of liquid.
 - c.** Accurate. Mix Y is the most apple because it has the greatest ratio of concentrate to water.
 - d.** Not accurate. The taste is determined by the ratio of concentrate to water. Since Mix Y has more concentrate per cup of water, it will have the most apple taste.
- 12.**
- a.** 6 : 4 (or 3 : 2)
 - b.** 2 : 3
 - c.** Not possible. This is discussing difference and to make a ratio, one would also have to know one of the amounts. Differences can be the same even when ratios between two quantities are different.
- 13.**
- a.** 4 : 1, part-to-part
 - b.** 1 : 5, part-to-whole
 - c.** 4 : 5, part-to-whole
- 14.**
- a.** Ratio of concentrate to juice: $\frac{12}{60} = \frac{1}{5}$
 - b.** Ratio of concentrate to water: $\frac{3}{12} = \frac{1}{4}$
 - c.** Ratio of water to juice: $\frac{2}{2\frac{1}{2}} = \frac{4}{5}$
 - d.** This fraction does not represent a ratio from this situation.
- 15.**
- a.** Less concentrated (less grapefruity); Using the original mix, and scaling by 3, 3 cans concentrate should be mixed with 12 cans of water. So, 15 cans of water makes the mix more watered down.
 - b.** The same concentration as the original; As stated in part (a), 12 cans of water plus 3 cans of concentrate would give 15 cans of juice.
 - c.** More concentrated (more grapefruity); Using the original mix and scaling by 2.5, 10 cans of water should be mixed with 2.5 cans of concentrate. Students might say the same concentration if they are mistakenly thinking of scaling as additive, that is, adding 6 cans of water and 6 cans of concentrate will give the same concentration. If students say this, you can point out that the ratio of concentrate to water in Mix C is over $\frac{1}{2}$, but the original is less than $\frac{1}{2}$.
 - d.** Less concentrated (less grapefruity); Using a scale factor of $\frac{1}{4}$, $\frac{1}{4}$ can of concentrate should be mixed with 1 can of water.
- 16.** The mixture will be the same as the original. Because the original two mixtures were the same ratio as the original mixing instructions, adding these two batches together will result in the same ratio. Interestingly, fractional notation may cause some difficulty if students consider this problem as $\frac{1}{4} + \frac{1}{4}$ instead of $\frac{(10 + 8)}{(32 + 40)} = \frac{1}{4}$.
- 17.**
- a.** Scale Factor is 24;
 $4 \times 24 = 96$ cans of water.
 - b.** Scale Factor is 5;
 $24 \times 5 = 120$ cans of juice.
 - c.** Scale Factor is $\frac{24}{5} = 4.8$;
 $4.8 \times 4 = 19.2$ cans of water.
 - d.** Scale Factor is $\frac{24}{5} = 4.8$;
 $4.8 \times 1 = 4.8$ or $4\frac{4}{5}$ cans of water.

- 18.** Raina's strategy works: $32 : 128$ is equivalent to $1 : 4$.
 Amelia's strategy works: She is simply applying a scale factor of 2 at each step.
 Krista's strategy is incorrect. This fraction would be $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64} \neq \frac{1}{4}$.
- 19. a.** 6 miles; Using equivalent ratios, $\frac{15}{1} = \frac{90}{x}$. The scale factor is 6.
b. About 4.3 miles; The scale factor is $\frac{13}{3}$, or 4.333...
- 20. a.** 6.5 miles; Using equivalent ratios, $\frac{0.25}{1} = \frac{x}{26}$. The scale factor is 26.
b. 20 miles; Using equivalent ratios, $\frac{0.25}{1} = \frac{5}{x}$. The scale factor is 20.
- 21.** About 2,000,000; Using equivalent fractions, $\frac{2}{1,000} = \frac{x}{1,000,000,000}$. The scale factor is 1 million.
- 22. a.** \$6; $\frac{\$18}{12 \text{ dozen}} = \frac{x}{4 \text{ dozen}}$. The scale factor is $\frac{1}{3}$. $18 \times \frac{1}{3} = 6$.
b. \$75; $\frac{\$18}{12 \text{ dozen}} = \frac{x}{50 \text{ dozen}}$. The scale factor is $\frac{25}{6}$. $18 \times \frac{25}{6} = 75$.
c. 18 dozen; $\frac{\$18}{12 \text{ dozen}} = \frac{\$27}{x}$. The scale factor is 1.5. $12 \times 1.5 = 18$.
d. 42 dozen; $\frac{\$18}{12 \text{ dozen}} = \frac{\$63}{x}$. The scale factor is 3.5. $12 \times 3.5 = 42$.
- 23. a.** 40; Using equivalent fractions, $\frac{10}{15} = \frac{x}{60}$. The scale factor is 4.
b. About 53.3, or 53; The scale factor is about 5.3.
c. 45 shots; Using equivalent fractions, $\frac{10}{15} = \frac{30}{x}$. The scale factor is 3.
d. About 68 shots; Using equivalent fractions, $\frac{10}{15} = \frac{45}{x}$. The scale factor is 4.5.
- 24.** 15.625; $12.5 \div 0.8 = x$; $x = 15.625$
- 25.** 10; $15 \times 2 = 30$; $20 \div 2 = 10$; $x = 10$
- 26.** 81; $4.5 \times 18 = 81$; $x = 81$
- 27.** 22.6; $15.8 \div 0.7 \approx 22.6$; $x \approx 22.6$
- 28.** 980; $245 \div 0.25 = 980$; $x = 980$
- 29.** 4; $4.5 \div 18 = 0.25$; $1 \div 0.25 = 4$; $x = 4$
- 30.** 2; $960 \div 48 = 20$; $0.1 \times 20 = 2$; $x = 2$
- 31.** 210; $900 \div 15 = 60$; $3.5 \times 60 = 210$; $x = 210$
- 32.** D; 10.3 (10) from North, 6.3 (6) from Central, and 3.4 (4) from South. The total from all schools is 1,200. The fraction of North to total is $\frac{618}{1,200}$, of Central is $\frac{378}{1,200}$, and of South is $\frac{204}{1,200}$. Using equivalent fractions, $\frac{618}{1,200} = \frac{x}{20}$. The scale factor is $\frac{1}{60}$. $618 \times \frac{1}{60} = 10.3$, $378 \times \frac{1}{60} = 6.3$, and $204 \times \frac{1}{60} = 3.4$. **Note:** Students might round down the 3.4 to 3, but 3.4 is closer to 4 than 2, which is the choice in A, B, and C.

Connections

- 33. a.** The ratio of the lengths of the top sides of the two Grumps is 0.8 to 1.2 or 2 to 3.
b. Since they are similar, any side of the small Grump is $\frac{2}{3}$ the length of the corresponding side of the larger Grump.
c. The top side of the small Grump is about 67% of the length of the top side of the larger Grump.
d. The scale factor from the small Grump to the large Grump is 1.5.
- 34. a.** The ratio of the areas of the two Grumps is 4 to 9.
b. The area of the smaller Grump is $\frac{4}{9}$ the area of the larger Grump.
c. The area of the smaller Grump is about 44% the area of the larger Grump.
d. The area scale factor from the small Grump to the large Grump is 2.25.

35. 0.93 in.; Possible explanations: The scale factor is 1.5. Therefore, $1.4 \div 1.5 \approx 0.93$; or the scale factor is $\frac{2}{3}$; $\frac{2}{3}$ of $1.4 \approx 0.93$.

36. B (0.6 times the scale factor of 1.5 equals 0.9.)

37. No. It means that for every 7 people that responded to the survey, 4 of them watched the Super Bowl. If the survey only sampled 7 people, then 4 of them watched. But if the survey sampled 7,000 people, about 4,000 watched. Four sevenths means that $\frac{4}{7}$ of the number sampled, whatever that number is, watched. For example, if the survey sampled 100 people, $\frac{4}{7} \times 100$ would be 57.14.

38. a. One piece will be 1.5 inches, and the other will be 3.5 inches. A ratio of 3 : 7 also means that one piece will be 0.3 of the fruit bar and the other piece will be 0.7 of the fruit bar. Thus, $0.3 \times 5 = 1.5$ and $0.7 \times 5 = 3.5$.

b. One piece will be 3 inches long, and the other will be 2 inches long ($60\% = 0.6$, $0.6 \times 5 = 3$).

c. One piece will be 3 inches long, and the other will be 2 inches long.

39. The 3 in the numerator in part (a) and the 60% in part (b) each represent a part; the 5 inches in the problem text and the 10 in the denominator in part (a) represent a whole; and 1 inch in part (c) represents the difference between parts.

For the Teacher Discuss what techniques students used to arrive at each of the answers. Which part was easiest to answer? Which way of phrasing the question (in terms of fractions, ratios, percents, differences) made the most sense for solving these problems?

40. a. 25% red paint

b. 28.6% red paint and 71.4% white paint

41. a. $\frac{3}{10}$ peanuts

b. $\frac{1}{8}$ almonds and $\frac{7}{8}$ other nuts

42. a. 9

b. 11, or any number greater than 10

c. 13, or any number greater than 12

d. 4.5, or any number greater than 4.5

e. 9, or any number greater than 9

f. 28

43. a. About 39.9% or 40%

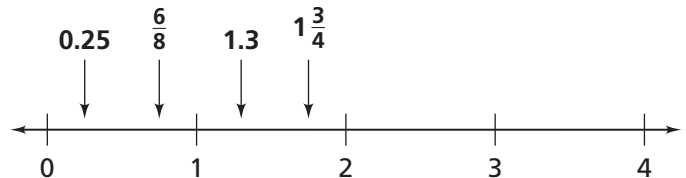
b. About 22%

c. The ratio of 12- to 17-year-olds who walk for exercise to 55- to 64-year-olds who walk for exercise is 5,520 to 12,595, or about 11 to 25.

d. The ratio of the percentage of 12- to 17-year-olds who walk for exercise to the percent of 55- to 64-year-olds who walk for exercise is 11 to 20.

e. Percents, because the number sampled in each category is not the same number, therefore percents seem more appropriate to use so that the two categories can be compared, based on numbers out of 100.

44.



For the Teacher Discuss how students are representing the numbers to place on the number line. Are they changing the way they are represented in the problem to a consistent form, such as all fractions or all decimals, etc.? What seems to be a natural way to begin dividing the segments on the number line?

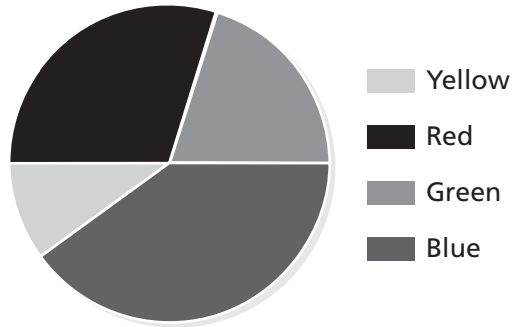
45. Possible answer: $\frac{3}{4} > \frac{2}{3}$, $\frac{3}{4}$ is greater because it is closer to 1. Its decimal equivalent is 0.75 as compared to about 0.67, the decimal approximation of $\frac{2}{3}$.

46. Possible answer: $\frac{1}{2} > 0.25$ ($0.5 > 0.25$).

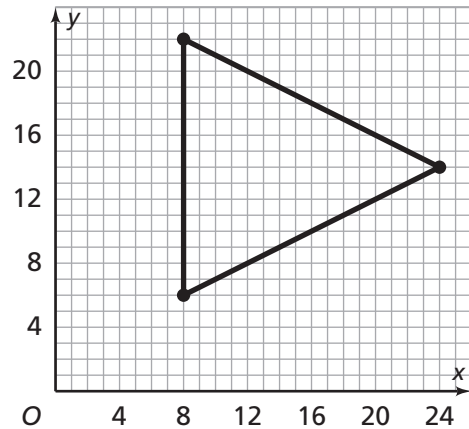
47. $\frac{4}{5} < \frac{11}{12}$ ($0.8 < 0.916\dots$)

48. $\frac{14}{21} = \frac{10}{15}$ ($\frac{2}{3} = \frac{2}{3}$)
49. $\frac{7}{9} > \frac{3}{4}$ ($0.78 > 0.75$ or $\frac{28}{36} > \frac{27}{36}$)
50. $2.5 > 0.259$ (2.5 is greater than 1, and 0.259 is less than 1.)
51. $30.17 > 30.018$ (Because 30 is the same in both, compare the tenths place; $1 > 0$, so $30.17 > 30.018$.)
52. $0.006 = 0.0060$ (Because the first three decimal places are the same in both, compare the next decimal place. The unwritten 0 in 0.006 equals the 0 in 0.0060, so $0.006 = 0.0060$.)
53. $0.45 = \frac{9}{20}$ ($0.45 = 0.45$, or $\frac{9}{20} = \frac{9}{20}$)
54. $1\frac{3}{4} > 1.5$ ($\frac{7}{4} > \frac{6}{4}$, or $1.75 > 1.5$)
55. $\frac{1}{4} < 1.3$ ($\frac{1}{4}$ is less than 1, and 1.3 is greater than 1.)
56. No. It means that 90%, or every 9 out of 10 people in the stadium, were between 25 and 55. There could have been 25,000 people in the stadium, in which case 22,500 would have been between the ages of 25 and 55 ($25,000 \times 0.9 = 22,500$). However, if there were only 100 people, then Alicia would be right that 90 were between those ages. Percents put actual numbers into a number that means "out of 100" in order to give a means of comparison.
57. H
58. A
59. a. 9; The scale factor is 3 ($12 \div 4 = 3$ and $3 \times 3 = 9$).
- b. 10; The numerator must be greater than 9 because $\frac{9}{12} = \frac{3}{4}$.
- c. 8; $\frac{9}{12} = \frac{3}{4}$, so the numerator must be less than 9.
- d. 16; The scale factor is $\frac{4}{3}$ ($12 \div 9 = \frac{4}{3}$ and $\frac{4}{3} \times 12 = 16$).
60. a. $\frac{6}{8} = \frac{9}{12} = \frac{12}{16}$
- b. $\frac{6}{9} = \frac{8}{12} = \frac{14}{21}$
- c. $\frac{4}{60} = \frac{5}{75} = \frac{6}{90}$
- d. $\frac{6}{4} = \frac{15}{10} = \frac{24}{16}$

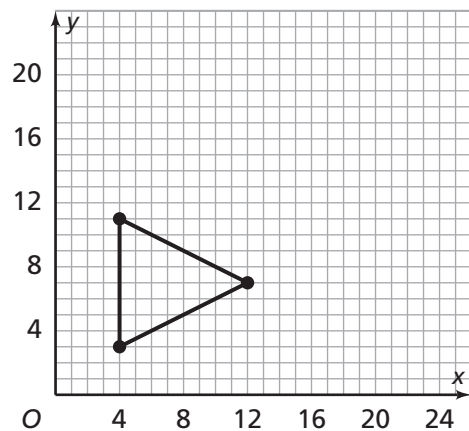
61. B
- 62.



63. a.



- b.



- c. The lengths are in proportion. The scale factor from the small triangle to the large triangle is 2 (or the scale factor from the large triangle to the small triangle is 0.5).
- d. The area of the small triangle is 25% of the area of the large triangle.
- e. The area of the large triangle is 400% of the area of the small triangle.

64. a. $BC \approx 3.42$. Possible strategies: $\frac{BC}{4} = \frac{6}{7}$.
 $BC = \frac{6}{7} \times 4 \approx 3.42$. $\frac{7}{6} = \frac{4}{BC}$. The scale factor is about 0.57. $0.57 \times 6 = 3.42$.
- b. $RU = 3.5$. Possible strategies: $\frac{RU}{7} = \frac{2}{4}$.
 $RU = \frac{2}{4} \times 7 = 3.5$. $\frac{4}{2} = \frac{7}{RU}$. The scale factor is 1.75. $1.75 \times 2 = 3.5$.
- c. $CD \approx 1.14$. Possible strategies: $\frac{CD}{4} = \frac{2}{7}$.
 $CD = \frac{2}{7} \times 4 \approx 1.14$. The scale factor is about 0.57. $0.57 \times 2 = 1.14$.

65. a. They most likely came up with the segments AB , BC , and DE by measuring. They could have used measuring tools or instruments to determine the length of segments they made. They probably staked off two points and measured the distance between them.
- b. By using equivalent ratios, based on the fact that the triangles are similar (triangle ADE is similar to triangle ABC). For example, if we compare corresponding sides of the large to the small triangle, we get $650 : 325$ as $AD : 300$ or $\frac{650}{325} = \frac{AD}{300}$. $\frac{650}{325} = \frac{2}{1} = \frac{AD}{300}$, so $AD = 600$. If we subtract AB from 600, we know that $BD = 300$.

Extensions

66. Possible answer: About 67% of dentists recommend sugarless gum to their patients who chew gum. 2 out of 3 dentists recommend sugarless gum to their patients who chew gum.
67. a. In 2002, about 61% of money spent on food was spent on food eaten at home. 39% was spent on food eaten away from home. (The total amount of money spent on food in 2002 was \$766,874,000,000). In 2010, about 58% of money spent on food was for food eaten at home. 42% was spent on food eaten away from home.
- b. The amount of money spent on food eaten away from home is increasing in relation to the total amount spent on food. 39% was spent on food eaten away from home in 2002 as compared to 42% in 2010.
- Note:** You may want to discuss this chart further with your students. Explain that students can use the significant, nonzero digits as the basis for comparison instead of the entire numbers.
68. a. Histogram B uses larger intervals, so more households fit in each interval and the bars go higher. Histogram B is slightly more uniform on the lower end, while Histogram A overall contains more gaps and is not as uniform.
- b. Possible answers: In Histogram A, the data seem to clump from 180 to 250 gallons, and in Histogram B, the data seem to clump from 160 to 260.
69. Television
70. Newspapers: About 34%; Magazines: About 5%; Television: About 20%; Radio: About 5%; Yellow Pages: About 5%; Internet: About 2%; Direct Mail: About 14%; Other: About 14%
- Note:** You may want to allow your students to use calculators for this Exercise, as the figures are rather large to calculate by hand.
71. Newspapers: About 17%; Magazines: About 7%; Television: About 33%; Radio: About 10%; Yellow Pages: About 1%; Internet: About 7%; Direct Mail: About 8%; Other: About 18%
- Note:** You may want to allow your students to use calculators for this Exercise, as the figures are rather large to calculate by hand.

72. Possible answers:

Overall, the percent spent on advertising in print media has decreased over the 10-year span from 2000 to 2010.

The percent spent on magazine advertising increased slightly over 10 years.

The greatest difference in spending over the 10 years was in television.

The least difference in spending over the 10 years was in magazines.

The greatest percent change in spending was in newspapers, down to 17% from 34%.

73. Possible responses: Students might argue that percents will show that the Internet is less expensive than radio. Students might also argue that the Internet has the potential to reach many people outside the typical listening range of radio stations.

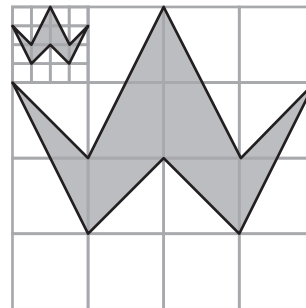
74. Percents are easily understood and often used to discuss trends over time. In this case, they would indicate the relative consistency of expenditures per medium. The differences would highlight the impressive overall dollar increase in advertising. The differences would also make a better headline. However, the trends in advertising would be more accurately represented by using percents.

For the Teacher Discuss how such big differences can exist in terms of actual expenditures while percents can remain relatively unchanged.

75. 500; Using equivalent fractions, $\frac{2}{50} = \frac{20}{x}$. The scale factor is 10.

76. About 1,500 beans; Using equivalent fractions, $\frac{x}{100} = \frac{30}{2}$. The scale factor is 50.

- 77. a.** Answers will vary. Sample answer: If you think of the small figure as the original, then the large figure is the image drawn on grid paper which scales the original up by a scale factor of 4.
- b.** Answers will vary. If you think of the large figure as the original, then the small figure is the image drawn on grid paper which scales the original down by a factor of $\frac{1}{4}$.



Note: Other scale factors could be used.

- c.** The perimeter of the similar figures can be found by multiplying the original scale factor by the corresponding scale factor of either the enlargement or the reduction. In the above example, the scale factor for the perimeter of the enlargement is 4 and the scale factor for the perimeter of the reduction is $\frac{1}{4}$. The area of the two similar figures is found by multiplying the area of one figure by the square of the scale factor to determine the area of the other similar figure. In the example above, the scale factor for the area of the enlargement is 4^2 and the area for the reduced figure is $(\frac{1}{4})^2$ or $\frac{1}{16}$.

78. a. The number of representatives from each state is determined by the ratio of the population of the state to the population in the United States. Therefore, the greater the population of a state, the more representatives that state will have. **Note:** There is a minimum number of representatives so small states are still better represented proportionately than large states.
- b. The number of senators is the same for every state, regardless of size or population. It is 2 per state.
- c. With the same number for every state, small states can get an equal say/voice/vote, in terms of the Senate. However, with the method of the House of Representatives, the large states get more representation or voice, thus the Congress would be reflecting the voice of the people.