Applications

1. **a.** Rectangles A and B are similar because the ratio of 2 to 4 (short side to long side) is equal to the ratio of 3 to 6 (short side to long side). Also, the scale factor is constant: 1.5.

   Parallelograms D and F are similar because the ratio of 2.75 to 3.5 (short side to long side) is equal to the ratio of 5.5 to 7 (short side to long side), and the corresponding angles are the same measure. Also, the scale factor is constant: 2.

   b. Answers may vary. Sample answers:

   Rectangle A: $\frac{2}{4} = 0.5$;

   Rectangle B: $\frac{3}{6} = 0.5$

   Parallelogram D: $\frac{2.75}{3.5} \approx 0.786$;

   Parallelogram F: $\frac{5.5}{7} \approx 0.786$

   The ratios for A and B are equivalent; also, the ratios for D and F are equivalent.

   c. The scale factor from A to B is 1.5. The scale factor from D to F is 2. Each of these scale factors is different from the ratios found in part (b).

   The scale factor compares a side in one figure to its corresponding side in a similar figure. The scale factor has to be constant from one shape to the other for any pair of corresponding sides. The ratio of adjacent side lengths describes the relationship between two measures within one shape. The ratio in one shape has to be equivalent to the ratio of corresponding sides in the similar shape.

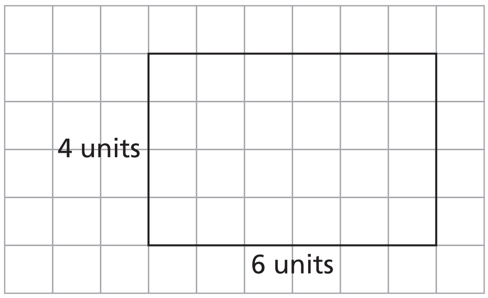
2. **a.** Answers will vary. Possible answer: (See Figure 1.)

   b. Answers will vary. For the sketch above, the smaller rectangle's ratio is 3 to 2; the larger rectangle's ratio is 7.5 to 5.

   Note: This assumes that the length is the longer dimension.

   The answer varies depending on the dimensions of the rectangles drawn.

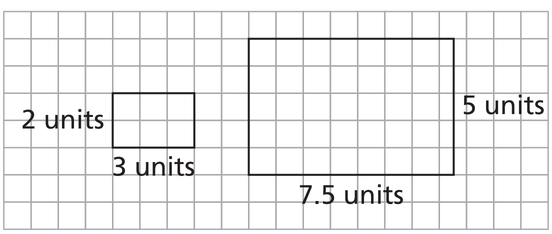
   c. Answers will vary. Possible answer for the sketch above is drawn here. The scale factors from this rectangle to the rectangles in part (a) are $\frac{1}{2}$ and $1\frac{1}{4}$.



   d. Answers will vary. Ratios of length to width are equivalent in all similar rectangles, so the answer should be equivalent to the answer for part (b).

   e. The length-to-width ratios of the three rectangles are equivalent. If you were to sketch another rectangle similar to the first three, the length-to-width ratio of that rectangle would be equivalent as well. Since there is a common scale factor for all corresponding side lengths, both parts of the length-to-width ratio would grow by the same amount. This would make the new ratio equivalent to the previous three.

Figure 1



3. a. A and B are similar. C and D are similar. They are similar because their corresponding angle measures are congruent. Also, each ratio of adjacent side lengths within one figure is equivalent to the ratio of corresponding side lengths in the similar figure. Last, scale factors from each side length in one figure to the corresponding side length in the similar figure are constant.

   b. The side-length ratios for Triangle A are 3 to 4, 3 to 6.5, and 4 to 6.5.

   The corresponding side-length ratios for Triangle B are 1.5 to 2, 1.5 to 3.25, and 2 to 3.25.

   When simplified, the ratios for Triangle A and Triangle B are equivalent: $\frac{3}{4} = \frac{1.5}{2}$, $\frac{3}{6.5} = \frac{6}{13} = \frac{1.5}{3.25}$, and $\frac{4}{6.5} = \frac{8}{13} = \frac{2}{3.25}$.

   The side-length ratios for Triangle C are 3 to 5, 3 to 6, and 5 to 6.

   The corresponding side-length ratios for Triangle D are 1.5 to 2.5, 1.5 to 3, and 2.5 to 3.

   When simplified, the ratios for Triangle C and Triangle D are equivalent: $\frac{3}{5} = \frac{1.5}{2.5}$, $\frac{3}{6} = \frac{1.5}{3}$, and $\frac{5}{6} = \frac{2.5}{3}$.

   c. Possible answer: The scale factor from A to B is $\frac{1}{2}$. The scale factor from C to D is $\frac{1}{2}$. The scale factors of these similar triangles tell how many times as great the corresponding side lengths or perimeter are from one figure to a similar figure. The ratio of adjacent side lengths within one triangle tells how many times as great one side length of the triangle is to another side length in the same triangle.

   d. In similar triangles, corresponding angles are congruent. This is different from corresponding side lengths because corresponding side lengths vary by a consistent scale factor.

4. 2.5 cm; Possible explanation: The scale factor from large triangle to the small triangle is 0.5. So, $a = 5 \times 0.5 = 2.5$.

5. 3 cm; Possible explanation: The ratio of 10.5 to 7 in decimal form is 1.5. Therefore, the ratio of b to 2 should also be 1.5. So, $b = 2 \times 1.5 = 3$.

6. $c = 60°$; Possible explanation: Angle c of the large triangle corresponds to the angle of the small triangle that measures 60°. Since corresponding angle measures of similar triangles are the same, c must be 60°.

   Note: All equilateral triangles are similar because the angles in an equilateral triangle are all 60°. Also, for all equilateral triangles, each ratio of adjacent sides is 1.

7. $d \approx 16.7$; Possible explanation: $\frac{10}{3} = \frac{d}{5}$, so $d = \frac{50}{3} \approx 16.7$

8. B

9. 0.25

10. The area of A is 15 square centimeters. The area of B is 240 square centimeters. The area of Rectangle B is 16, or 4^2 (the square of the scale factor), times the area of Rectangle A.

11. a. $x = 2$inches

    b. 0.5

    c. The area of C is 16 square inches. The area of D is 4 square inches. The area of D is $\frac{1}{4}$ the area of C. The factor $\frac{1}{4}$ is found by taking the square of the scale factor, $\frac{1}{2} : \frac{1}{4} = \left(\frac{1}{2}\right)^2$.

12. a. 108 square feet, or 12 square yards.

    b. $264

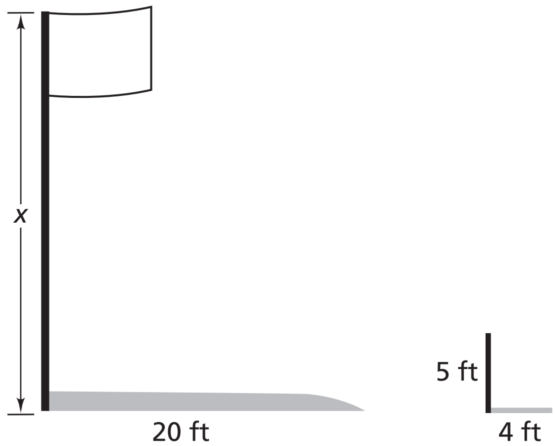
13. a. 22.5 feet by 30 feet (or 7.5 yards by 10 yards); The dimensions of the library are 2.5 times the corresponding dimensions of the bedroom.

    b. 675 square feet, or 75 square yards.

    c. $1,650

14. 556 feet; sample sketch: (See Figure 2.)

15. 25 feet; sample sketch:



16. a. One way is to determine the scale factor between Greg's image in the picture and Greg's actual height. For example, if Greg is 1 inch tall in the picture and is 5 feet (or 60 inches) tall in real life, the scale factor from picture to real life is $\frac{60}{1}$. You can then measure the height of the building in the picture and multiply that height by 60 to find the actual height of the building.

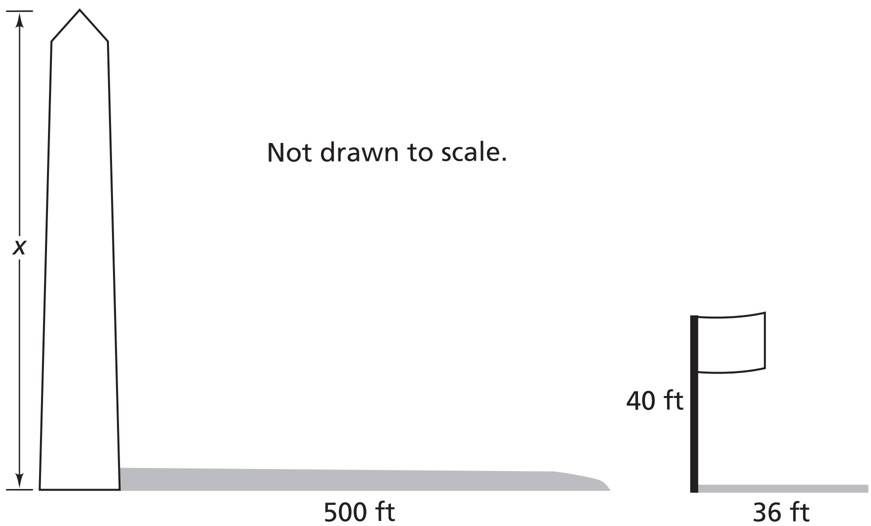
b. 1.25 inches; Since Greg is 5 feet tall in real life and is $\frac{1}{4}$ (or 0.25) inch tall on the screen, the scale factor from real life to the picture is $\frac{0.25}{60}$. The building is 25 feet, or 300 inches. If you multiply that height by $\frac{0.25}{60}$, the result is the height of the building on the screen: 1.25 inches.

c. You need to take a picture with the tall object and another object that you know the actual height of. Then, you can determine the scale factor from the picture to the real object. To determine the scale factor, you need to divide one object's real height by the object's height in the picture. Then, measure the height of the tall object in the picture and multiply it by the scale factor.

17. Approximately 65 feet; solve for h in the proportion $\frac{16}{9} = \frac{115}{h}$; or, multiply 115 feet by the scale factor $\frac{9}{16}$, or 0.5625.

18. Taylor is right. The two triangles are similar, because if you convert the measures of Triangle B to inches, the scale factor from A to B is 24 for each side.

Figure 2



Connections

19. not equivalent

20. not equivalent

21. equivalent

22. equivalent

23. not equivalent

24. equivalent

25. Answers will vary.

26. Answers will vary. Sample answer: 10 to 6 and 15 to 9. In each answer, the division of the first number by the second should give the same result as the division of the first number in the question by the second number in the question.

27. Answers will vary. Sample answer: 8 to 2 and 20 to 5. In each answer, the division of the first number by the second should give the same result as the division of the first number in the question by the second number in the question.

28. Answers will vary. Sample answer: 12 to 28 and 7.5 to 17.5. In each answer, the division of the first number by the second should give the same result as the division of the first number in the question by the second number in the question.

29. Answers will vary. Sample answer: 3 to 2 and 0.75 to 0.5. In each answer, the division of the first number by the second should give the same result as the division of the first number in the question by the second number in the question.

30. a. 40 in. Answers will vary. Possible answer: The dog in the picture is 5 inches long. Since the picture of the dog is 12.5% of the real dog's size, then $0.125 \times$ the real length $=$ the picture's length, or 5 inches. You can rewrite this as $L = 5 \div 0.125$, or 40 in.

b. 23 in. Answers will vary. Possible answer: The dog in the picture is $2\frac{7}{8}$ in. tall. $2\frac{7}{8}$ is 0.125 times the real dog's height, so you can divide $2\frac{7}{8}$ by the scale factor, 0.125, to find the real height of the dog.

c. Duke is 8 times as large as the picture. Using 200% enlargement, you can double the size of the picture. Use the 200% enlargement three times in a row: $2 \times 2 \times 2 = 8$ times as large a picture.

31. a. $(0.5x, 0.5y)$

b. Yes, they are similar. The scale factor is 0.5 from triangle ABC to the new triangle.

32. a. 10 square units; 15 square units

b. 16,000 m^2; 24,000 m^2

33. G

34. a. Approximately 1.7 meters

Note: The triangles formed by the mirror and the heights of the student and the teacher are similar. The angles located at the mirror are congruent. Students can use their knowledge of similarity to find the teacher's height.

b. Yes; a meter is a little more than a yard, which is equal to 3 feet. The teacher is almost 2 meters tall, which means that she is a little less than 6 feet tall. This is a reasonable height for an adult.

35. a. Note: Decimal equivalents are approximations. $\frac{55}{60} = 0.92$; $\frac{60}{65} = 0.92$; $\frac{60}{63} = 0.95$; $\frac{48}{50} = 0.96$; $\frac{60}{58} = 1.03$; $\frac{65}{66} = 0.98$; $\frac{60}{60} = 1.0$; $\frac{67}{63} = 1.06$; $\frac{62}{67} = 0.93$; $\frac{70}{65} = 1.08$.

Answers may vary for patterns that are noticed. Possible answer: All the ratios are close to 1. All the numerators of the fraction forms are similar, and all of the denominators of the fraction forms are similar.

b. The mean is about 0.98.

c. About 60.76 in.; $\frac{\text{arm span}}{62}$ will be about 0.98, so arm span $= 62(0.98) = 60.76$ in.

36. a. complement: 70°, supplement: 160°

b. complement: 20°, supplement: 110°

c. complement: 45°, supplement: 135°

37. a. $\frac{1}{3}$

b. Possible answer: The ratio of 6 to 12 is equivalent to the ratio of x to 4. The ratio of 6 to x is equivalent to the ratio of 12 to 4.

c. $x = 2$ cm; the length of the short side of Rectangle A is half the length of the long side of Rectangle A. So, x must be half the length of 4 cm, which is 2 cm.

d. 9 : 1

38. C

39. a. M and Q are similar.

b. scale factor from Q to M: $\frac{2}{3}$;

scale factor from M to Q: $\frac{3}{2}$;

scale factor from L to N: $\frac{1}{2}$;

scale factor from N to L: 2.

c. M to Q: $\frac{9}{4}$; L to N: 4.

Extensions

40. a. $\frac{10}{8} = 1.25$

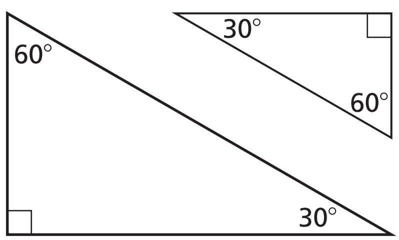
b. The ratio using the longest sides is $\frac{20}{16} = 1.25$. The same ratio is obtained using other sides as well. This ratio is the same as the scale factor in part (a).

c. $\frac{8}{10} = 0.8$

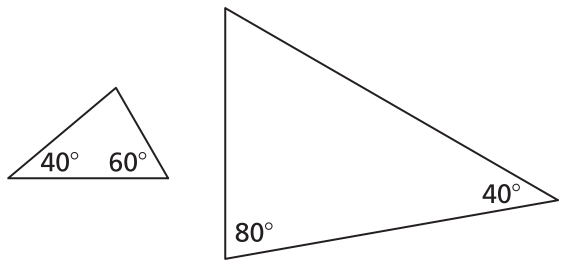
d. The ratio using the longest sides is $\frac{16}{20} = 0.8$. The same ratio would be obtained using other sides as well. This ratio is the same as the scale factor in part (c).

e. The scale factor tells by what factor each side is enlarged or reduced. The ratio of the corresponding sides is measuring the same quantity. Yes, the pattern will be true in general.

41. a. The drawings vary; however, all triangles with the given angles will be similar to each other.



b. The drawings vary; however, all triangles with the given angles will be similar to each other.

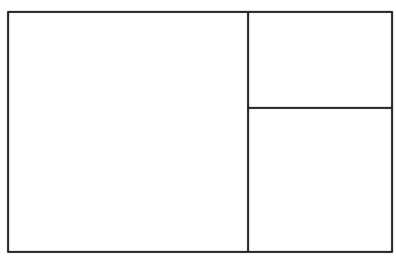


c. Conjecture: If the interior angle measures of a triangle are the same as those of another triangle, then the triangles are similar.

42. a. Rectangle A: ratio is 2.25 to 0.25

Rectangle B: ratio is 2 to 1.25, which gives 1.6 as a decimal number.

Rectangle C: ratio is 1 to 0.75

b. 

The smaller rectangle is a golden rectangle. The ratio of long side to short side is about 1.6.

43. a. Triangles A, C, and D are similar. The corresponding angle measures are congruent. Also, the ratios between the corresponding adjacent sides are the same.

For example: Triangle A's longest side to shortest side: $\frac{17}{30} \approx 0.57$; Triangle D's longest side to shortest side: $\frac{8.5}{15} \approx 0.57$. They are the same.

Triangle A's shortest side to middle side: $\frac{17}{21.6} \approx 0.79$; Triangle C's shortest side to middle side: $\frac{12.75}{16.2} \approx 0.79$. They are again the same.

Note: Students have to use the fact that the sum of the angles in a triangle are 180°.

b. Triangle A: $\frac{30}{12} = 2.5$

Triangle B: $\frac{10.4}{6} \approx 1.73$

Triangle C: $\frac{22.5}{9} = 2.5$

Triangle D: $\frac{15}{6} = 2.5$

Similar triangles have the same base-to-height ratio; the nonsimilar triangle does not share that ratio.

44. a. Yes, since $\frac{4}{6} = \frac{8}{12}$.

b. No, since $\frac{4}{6} \neq \frac{9}{11}$.

c. Yes, since $\frac{4}{6} = \frac{6}{9}$.

d. Yes, since $\frac{4}{6} = \frac{3}{4.5}$.

45. No. None of the given paper sizes have the same base-to-height ratio as the photo.

46. Use the 50% reduction two times in a row (i.e., copy the photo once, then take the image and make its copy again.) Each time the drawing will be reduced to half its size. So, after two reductions it will be $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of its original size.

47. (1) Applying a 50% reduction three times in a row using the image each time will reduce the size to 12.5% of its original dimensions. The area would be $\frac{1}{64}$ the area of the original.

(2) Apply a 60% reduction two times in a row to get a picture that is 36% of the original size. The area would be $\frac{81}{625}$ the area of the original.

48. You can enlarge the original 4 inch-by-6 inch picture until the side that is originally 4 inches is enlarged to 11 inches. The 6-inch side is then enlarged to 16.5 inches. This is the largest possible image to fit paper that is 11″ by 17″. This requires an enlargement of 275%.

Note: To accomplish this, first enlarge the original photograph to 200%, then enlarge the result a second time using 125% (the new copy is now 2.5 times larger than the original). Finally take this copy and enlarge it 110%. Because the scale factors are multiplicative (and therefore commutative) the order in which these enlargements are done does not matter.

49. a. Each number in the sequence is found by adding the previous two numbers. The following four numbers in the sequence will be: 610; 987; 1,597; 2,584.

b. The approximate decimal equivalents of the fractions in order are: 1, 2, 1.5, 1.67, 1.6, 1.625, 1.615, 1.619, 1.618, 1.618, 1.618, . . . (1.618 repeats). The sequence approaches a number that is very close to the estimation of the golden ratio in Exercise 44. (In fact, the "limit" of this sequence will be equal to the golden ratio.)

50. Francisco is right. All angles of a regular polygon are equal in measure, as are the sides. Since the measures of corresponding sides for any regular polygons with the same number of sides have the same ratio, a regular polygon of any size is similar to all other sizes of that regular polygon.

Thus, as a regular polygon, all squares are similar. The ratio of side lengths of any square is always equal to 1 : 1 : 1 : 1, and all angles are 90°.

Not all rectangles are similar because the ratios of the lengths of adjacent sides can be anything. So, some pairs of rectangles are not simple magnifications of each other.

Not all rhombi are similar because their angles can be different, even for rhombi with the same side lengths as each other.

51. Vernon is right. In an isosceles right triangle, the equal sides make the right angle and they have the ratio 1 : 1. All isosceles right triangles are similar because any isosceles right triangle has same 45°, 45°, and 90° angles, and the sides have the ratio $1 : 1 : \sqrt{2}$.