Applications

1. These conditions define a unique isosceles triangle.
2. These conditions define a unique equilateral triangle.
3. These conditions cannot be satisfied by a triangle, since $7 + 8 = 15$.
4. These conditions define a unique scalene triangle.
5. **a.** Exercise 2 is equilateral.
   b. Exercise 1 is isosceles.
   c. Exercise 4 is scalene.
   d. Exercise 1 and 2 have at least two angles of the same measure.
6. If all angles of a triangle are 60°, the triangle must be equilateral.
7. Any isosceles triangle has two angles the same size.
8. The two 3-foot poles and the 5-foot poles will make a tent, but it will be a low tent. The two 3-foot poles and the 6-foot pole and the two 3-foot poles and the 7-foot pole will not make a tent, because the sum of any two sides is not greater than their third side.
9. **a.** Two different triangles are possible.
   b. The triangle is unique by SAS.
   c. The triangle is unique by ASA.
   d. No triangle is possible because the sum of the measures of the angles exceeds 180°.
10. It is possible to build a quadrilateral with sides of the given lengths, but the shape is not unique. Rectangles, nonrectangular parallelograms, and kites are all possible, as are some nonsymmetric figures.
11. It is possible to build quadrilaterals with these side lengths, including an isosceles trapezoid.
12. Many rhombuses are possible, including a square.
13. Not possible; since $4 + 3 + 5 < 14$.
14. **a.** Exercise 12
    b. Exercise 10 and 12
    c. Exercise 10 and 12
    d. Exercise 10 and 11
15. Exercise 12
16. Exercise 11
17. **a.** Angles *b*, *e*, and *g* measure 35°; angles *a*, *c*, *d*, and *f* measure 145°.
    b. Vertical angle pairs are a and *c*; *b* and the 35° angle; *e* and *g*; and *d* and *f*.
18. The first (green) figure has only rotation symmetry (half turn); the second (yellow) figure has only reflection symmetry; the third (heart) figure has only reflection symmetry; the fourth (blue cross) figure has four lines of symmetry and rotational symmetry in multiples of 90°; the fifth (red) figure has three lines of symmetry and rotational symmetry in multiples of 120°.
19. C
20. H
21. C
22. F
23. Squares require all sides to be the same length.
24. Rectangles require opposite sides to be the same length, but not all four the same length.
25. No, that won't work because $5 + 5 = 10$, so the brace would force the rhombus to become simply a straight line.
26. You need to move at least two vertices to make a non-rectangular parallelogram.
27. You do not need to move any vertices because a non-rectangular parallelogram will have two opposite obtuse angles.

Connections

28. B

29. **a.** All three are parallelograms. This means that opposite sides are parallel and congruent; opposite angles are congruent; consecutive angles are supplementary.

b. Rectangle 2 does not have equal length sides, making it different from the square; parallelogram 3 does not have four right angles, making it different from both the square and the rectangle.

30. Angles 1, 3, and 5 are all 63°; angles 2 and 4 are both 117°.

31. The cross braces that turn quadrilaterals into linked triangles provide rigidity to the structures.

32. **a.** Triangles, parallelograms, dodecagons, 36-gons

b. The rug has 180° and 360° rotation symmetries. The 36-gons inside the rug also have 180° and 360° rotation symmetries.

33. **a.** Arizona's flag and New Mexico's flag have vertical lines of symmetry through the middle. Ohio's flag does not have a horizontal line of symmetry because the stars don't all match up. New Mexico's flag has a horizontal line of symmetry.

b. The circle in the Ohio flag, the stars in the Ohio and Arizona flags, and the design in the New Mexico flag all have rotation symmetry through the middle.

c. Answers will vary.

34. H

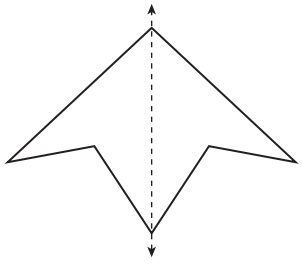
35. B

36. Answers will vary. Possible answers: Pattern A: squares, rectangles, triangles, and octagons. Pattern B: triangles, squares, rectangles, and octagons

37. Pattern A: The quilt has reflection symmetry around a vertical line in the center, a horizontal line in the middle, and two diagonal lines. Note: Some students may disagree since the colors may not match up, but if they focus on the shapes there is both reflection and rotation symmetry.

Pattern B: Again , disregarding color, the quilt is in the shape of a square and has a vertical line of symmetry, a horizontal line of symmetry, and two diagonal lines of symmetry. It also has 90°, 180°, 270°, and 360° rotation symmetries.

38.



Extensions

39. a. 180°

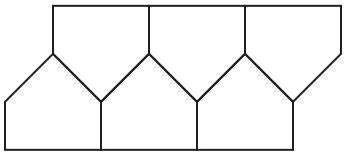
b. Both pairs of angles are on opposite sides of a transversal between parallel lines.

c. The angles 1, 2, and 3 have the same measures as angles 6, 2, and 5 respectively, and angles 6, 2, and 5 are the angles of a triangle. Since the sum of the measures of angles 1, 2, and 3 is the measure of a straight angle, the sum of the measures of those angles is 180°. This means that the sum of the measures of angles 6, 2, and 5 is also 180°.

40. a. If the sum of four sides of a proposed pentagon is less than the fifth side, it will be impossible to build a pentagon. For example, if the sides-to-be are 1, 1, 1, 1, and 5, it is impossible.

b. Pentagons are not rigid figures, a fact that can be seen readily with a polystrip experiment.

c. Regular pentagons cannot be used as tiles, but nonregular pentagons like the "home plates" pictured below can be used as tiles.



41. a. As you push on one crank, the driver, the opposite crank and the follower, are pushed outward and shift the coupler in two ways: down on the side of the driver and up on the side of the follower. The reverse is true when you pull on the crank.

b. To make a stirring mechanism, attach two spoons to *D* and *C*, perpendicular to the plane formed by *A*, *B*, *C*, and *D*. To make a wiping mechanism, attach wipers to *D* and *C* in the same plane as *A*, *B*, *C*, and *D*.

42. The figure shown is a model of a pantograph used to make enlargements of drawings.

a. Regardless of the strip held fixed, the figure will move in a way that keeps all parallelism relationships intact.

b. If you introduce a bracing strip between points *F* and *B*, the whole figure will become rigid.

43. a. The directions are inadequate.

b. The directions will define exactly the shape that is expected.

c. The directions will define exactly the shape that is expected.

44. a. Draw a square with sides 1 inch.

b. Draw a rectangle with sides 1.25 and 0.5 inches.

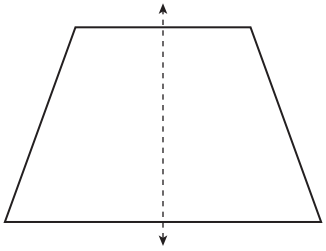
c. Draw a rhombus with sides 1 inch and one angle of 60°.

d. Draw a quadrilateral with $\overline{AB} = \frac{3}{4}$ in. $\angle B = 135°$. $\overline{BC} = 0.5$ in. $\angle C = 115°$. $\overline{CD} = 1\frac{1}{8}$ in. **Note:** Other correct directions are possible.

e. The quadrilateral has 4 sides, 4 angles, and 2 diagonals. A unique quadrilateral cannot be defined by 4 of the 10 elements. At least 5 elements are needed to draw a unique quadrilateral. **Note:** It is not possible to construct a unique quadrilateral with 1 side and 4 angles.

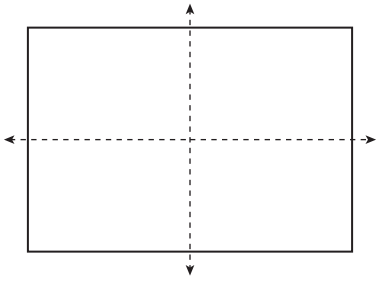
45. a. No. This is not a line of symmetry because the part of the figure on one side of the line does not look like it is being reflected in a mirror to form the part on the other side of the line.

b. vertical line through the middle of top and bottom sides



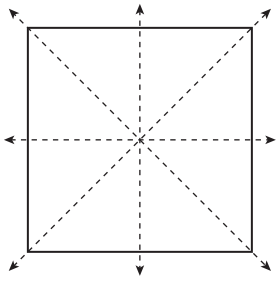
46. a. No. This is not a line of symmetry because the part of the figure on the side of it does not look like it is being reflected in a mirror to form the part on the other side of the line.

b. horizontal and vertical lines through the middle



47. a. Yes. This is a line of symmetry because the part of the figure on the one side of it does look like it is being reflected in a mirror to form the part on the other side of the line.

b. horizontal, vertical, and a diagonal line from right to left, all going through the center



48. a. Yes. This is a line of symmetry because the part of the figure on the one side of it does look like it is being reflected in a mirror to form the part on the other side of the line.

b. The rhombus has the other diagonal as a line of symmetry.

49. a. No. This is not a line of symmetry because the part of the figure on the one side of it does not look like it is being reflected in a mirror to form the part on the other side of the line.

b. The parallelogram has no lines of symmetry.