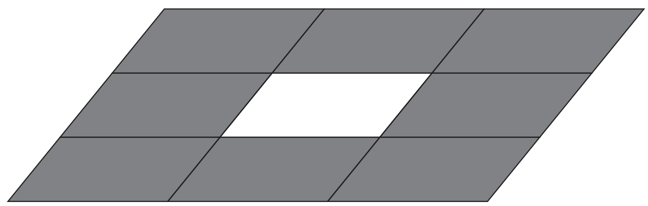
Applications

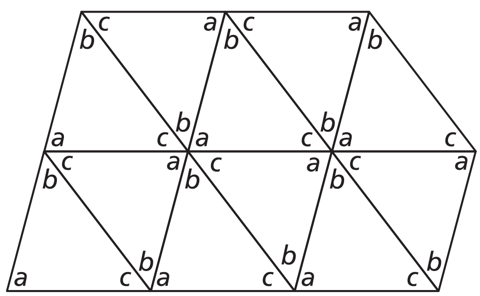
1. Interior angles of regular polygons.
   a. 120°   b. 150°
2. The length of sides does not affect angle measures in similar figures. This is a property of dilations that will be explored explicitly in the *Stretching and Shrinking* Unit that follows this Unit.
3. $x = 60°$
4. $x = 90°$
5. $x = 62°$
6. $x = 120°$
7. $x = 23°$
8. $x = 120°$
9. $x = 70°$
10. $x = 113°$
11. The sum of the acute angles is 90°. The triangle angle sum (180°) minus a right angle (90°) is 90°.
12. Dodecagon
    a. 1,800°
    b. 150°
    c. This figure will not tile a flat surface, because no integer multiple of 150 equals 360, the angle sum at each vertex of a tiling pattern.
13. Correct answer is B.
14. 360°
15. The answer to all questions is 360° because they will make exactly one complete turn as they skate around any polygon.

Connections

16. The interior angles of both add to 1,440°, assuming that the interior angles of the star are measured as rotations (allowing five angles of 36° and five angles of 252°). Dissection of the star will produce 10 triangles. All angles of those triangles will be used in some way as part of the angles of the star, except for 360° around the center point. Thus the angle sum should be 8(180) = 1,440.
17. $\angle ADC = \angle ADB$ and both are right angles; $\angle DAC = \angle DAB$ and both are 30° angles because the angles of an equilateral triangle must each be 60° to give a sum of 180° and symmetry implies bisection; $\angle CDA = \angle BDA$ and both are 60° angles because the angles of an equilateral triangle must each be 60°; $CD = DB = \frac{1}{2}\,CB$ due to symmetry.
18. Correct answer is G.
19. Copies of a parallelogram can be used as tiles, as shown in the following sketch:



20. Any scalene triangle can also be used as a tile.



21. The aim of this task is to get students to reflect on mathematical practices that provide justification for discoveries. There is something to be said for the strategies proposed as convincing by each group—empirical evidence is very often the first step toward a generalization and, for most people, it provides practical intuitive confidence in a result. The limitation is, of course, that drawing and measurement are inexact and they only give inductive evidence of what might be true. The thinking that infers an angle sum of 360° from imagining the turns that one makes in a complete circuit of a polygon is both intuitively appealing and logically convincing. The argument from a prior result about interior angles and deduction of connected exterior angles is perhaps more mathematically logical (assuming that the interior angle result is well established), but the path from the starting point to the result about exterior angles requires some fairly abstract and formal reasoning (sum of interior and exterior angles is $n(180)$; sum of interior angles is $180(n - 2)$; so sum of exterior angles is $2(180)$). This last reasoning is, of course, only as valid as the prior result.

Extensions

22. even-sided polygons

a. Following the pattern, the measure of an interior angle of a regular decagon would be $\frac{4}{5}$ of 180°, or 144°. For dodecagons (12 sides), the pattern predicts $\frac{5}{6}$ of 180°, or 150°. These entries are correct according to results from Problems 2.1 and 2.2.

Also, one could do an algebraic derivation from the formula $\frac{(n-2) \cdot 180}{n}$, but this is beyond what is reasonable to expect for even very capable CMP students at the start of Grade 7. Realizing that this pattern is observed for even numbers $n = 2k$, we get:

$$\frac{(n-2) \cdot 180}{2k} = \frac{(2k-2) \cdot 180}{2k}$$

$$= \frac{2(k-1) \cdot 180}{2k} = \frac{(k-1) \cdot 180}{k}$$

b. In the case of odd numbers $n = 2k + 1$, we get:

$$\frac{(n-2) \cdot 180}{n} = \frac{(2k+1-2) \cdot 180}{2k+1}$$

$$\frac{(2k-1) \cdot 180}{2k+1}$$

This leads to the pattern in the following table:

Regular Polygons

Number of Sides	Measure of Interior Angle
3	$\frac{1}{3}$ of 180° = 60°
5	$\frac{3}{5}$ of 180° = 108°
7	$\frac{5}{7}$ of 180° ≈ 128.57°
9	$\frac{7}{9}$ of 180° = 140°

23. The number 1,660 is not an integer multiple of 180, so it cannot be correct for the angle sum of any polygon.

24. Trevor's method of drawing diagonals from one vertex will work on the arrowhead figure but not the more irregular figure; Casey's method of connecting all vertices to a central point will also work on the arrowhead figure, but not the more complex figure.

What will work in both cases is finding a way to connect vertices that splits the figure into triangles. Each triangle will hold 180° of angle, and then you have to subtract angles that are not part of any original angle in the polygon. Applying this strategy to the figure on the left gives two triangles and an angle sum of 360°. Applying the strategy to the figure on the right gives six smaller triangles and an angle sum of 1,080°, which is what one expects for an octagon: (8 − 2)180°.

25. Counting diagonals.

a. Any quadrilateral has two diagonals; any pentagon has five diagonals.

b. Any hexagon has nine diagonals; any heptagon has 14 diagonals.

c. Looking for a pattern: (See Figure 1.)

d. $D = \frac{n^2 - 3n}{2}$

Figure 1

Number of Sides	4	5	6	7	8	9	10	11	12
Number of Diagonals	2	5	9	14	20	27	35	44	54